Sub-optimal Control Over AWGN Communication Network

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Abstract

In this paper a sub-optimal control technique is proposed for a linear Gaussian system with a few distributed interacting sub-systems. Controller of each sub-system has only access to its own measurement and a noisy version of measurement and control vectors of other sub-systems that are communicated through an Additive White Gaussian Noise (AWGN) communication network. The power to be allocated to each transmitter antenna is calculated so that the received signal is the transmitted signal plus additive white Gaussian noise. Under some conditions it is shown that the proposed sub-optimal control technique results in bounded mean square stability. The satisfactory performance of the proposed technique in stabilizing the dynamic system over communication channels with small noise is illustrated by computer simulations.

Keywords: Linear Gaussian system, AWGN communication network, sub-optimal control.

I. Introduction

A. Motivation and Background

In recent years, there has been an interest in replacing point to point wiring by wireless communication network for exchanging information between distributed sub-systems of a distributed controlled system as point to point wiring takes space, requires regular maintenance and it is costly. This motivates research on optimality and stability of dynamic systems over communication channels subject to imperfections (channel noise, distortion, etc.). Some results addressing basic problems in stability and optimality of dynamic systems over communication channels subject to imperfections can be found in [1]-[14].

Many systems have linear dynamics and dynamic systems can be viewed as continuous alphabet information sources with memory. Therefore, many works in the literature (e.g., [1],

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[12],[13],[14]) are dedicated to the question of optimality and stability over Additive White Gaussian Noise (AWGN) channel, which itself is naturally a continuous alphabet channel. In [12], the authors presented a sub-optimal decentralized control technique for bounded mean square stability of a large scale system with cascaded clusters of sub-systems. Each sub-system is linear and time-invariant and both sub-system and its measurement are subject to Gaussian noise. The control signals are exchanged between sub-systems without any imperfections, but the measurements are exchanged via an AWGN communication network. In [13], the authors presented an optimal control technique for asymptotic bounded mean square stability of a partially observed discrete time linear Gaussian system over AWGN channel. Here, again authors assumed that control signal is transmitted without any imperfections. In [14], the authors addressed the continuous time version of the problem addressed in [13].

B. *Paper Contributions*

In this paper a sub-optimal control technique is proposed for a linear Gaussian system with a few distributed interacting sub-systems. Controller of each sub-system has only access to its own measurement and a noisy version of measurement as well as control vectors of other sub-systems that are communicated by an AWGN communication network. Exchange of measurement and control vectors via AWGN channels results in non-classical and different information pattern for each sub-system. This is due to channel noise. As is shown in [15], the optimal stochastic solution for this problem is a nonlinear strategy. Nevertheless, in this paper we present a sub-optimal but linear decentralized control solution for this problem. For a distributed system with n interacting sub-systems, this linear sub-optimal solution is obtained by solving n separated centralized LQG problems. Solution to each separated LQG problem defines the control strategy for a sub-system (let us say sub-system i), in which in this strategy, the noisy version of the control vectors of other sub-systems (let us say $j \neq i$) communicated via AWGN channels to sub-system i are used. Similarly, in the associated Kalman filter equation, the noisy version of the measurement vectors of other sub-systems communicated via AWGN channels are used. The power to be allocated to each transmitter antenna is calculated so that the received signal is the transmitted signal plus additive white Gaussian noise. Under some conditions it is shown that the proposed sub-optimal control technique results in bounded mean square stability. The satisfactory performance of the proposed technique in stabilizing the dynamic system over communication channels with small noise is illustrated by computer simulations. Non-large scale version of the system considered in [12] includes only a cluster of sub-systems with a few distributed interacting sub-systems, which is exactly the kind of system considered in this paper. Note that [12] is concerned with the case when only measurement vectors are exchanged between sub-systems via AWGN channels; while in the problem considered in this paper, control vectors are also exchanged between sub-systems via AWGN channels. Note also that [12] uses a coding technique to match transmitted measurement vectors to AWGN channels. This paper also extends the results of [13] to the cases with multiple systems and when the transmission of control vectors are also subject to communication imperfections.

C. Paper Organization

The paper is organized as follows. In Section II, the problem formulation is presented. Section III is devoted to the sub-optimal control technique. Section IV is devoted to the stability result. Simulation results are given in Section V, and in Section VI the paper is concluded by summarizing the contributions of the paper and direction for future research.

II. Problem Formulation

Throughout, certain conventions are used. $\mathbb{R}$ denotes the space of real numbers, $\rho(\cdot)$ the spectrum norm and $\doteq$ means 'by definition is equivalent to'. $N(m,n)$ denotes the Gaussian distribution with mean m and variance n, $cov(k)$ denotes the covariance of the random variable k and $\mathcal{E}[k]$ is the expected value. A^{-1} denotes the inverse of square matrix A and V' is the transpose of matrix/vector V. $diag(\cdot)$ denotes the block diagonal matrix and I_n denotes the identity matrix with dimension $n \times n$.

This paper is concerned with the sub-optimality and stability of a linear Gaussian system with a few distributed sub-systems over AWGN communication network. The dynamic system, communication network and cost functional are described below:

Dynamic system: The dynamic system is a linear Gaussian system with a few, let us say n distributed sub-systems S_i, $i=\{1,2,...,n\}$, with the following representation:

$$S_i : \begin{cases} x_{t+1}^{(i)} = \sum_{j=1}^{n} A_{ij} x_t^{(j)} + \sum_{j=1}^{n} B_{ij} u_t^{(j)} + w_t^{(i)}, \\ y_t^{(i)} = C_i x_t^{(i)} + v_t^{(i)}. \end{cases} \tag{1}$$

Here, $x_t^{(i)} \in \mathbb{R}^{n_i}$, $u_t^{(i)} \in \mathbb{R}^{m_i}$, $y_t^{(i)} \in \mathbb{R}^{l_i}$, $w_t^{(i)} \in \mathbb{R}^{n_i}$, $v_t^{(i)} \in \mathbb{R}^{l_i}$, $w_t^{(i)}$ i.i.d. $\sim N(0, \Sigma_w^{(i)})$, $v_t^{(i)}$ i.i.d. $\sim N(0, \Sigma_v^{(i)})$ and $x_0^{(i)} \sim N(\bar{x}_0^{(i)}, \bar{v}_0^{(i)})$. Throughout, it is assumed that the matrices A_{ij}, $B_{ij}, C_i, \Sigma_w^{(i)}$, $\Sigma_v^{(i)}$, $\bar{x}_0^{(i)}$, $\bar{v}_0^{(i)}$ are known and $\{x_0^{(i)}, w_t^{(j)}, v_t^{(k)}\}_{i,j,k=1}^{n}$ are mutually independent.

Communication network: Each sub-system S_i broadcasts each component of its control vector

$u_t^{(i)}$ and measurement vector $y_t^{(i)}$ by an AWGN communication channel to all other sub-systems ($j \neq i$). This network has the following representation:

$$\tilde{z}_t^{(ij)} = \alpha^{(ij)} z_t^{(i)} + n_t,\ \ \tilde{z}_t^{(ij)}, z_t^{(i)}, n_t \in \mathbb{R},\ \ E[z_t^{(i)}]^2 \leq p^{(i)}, i, j \in \{1, 2, ..., n\}, i \neq j, \tag{2}$$

where $z_t^{(i)}$ is the channel input (a component of the control vector $u_t^{(i)}$ or measurement vector $y_t^{(i)}$), $\alpha^{(ij)}$ is the known attenuation factor in communication from sub-system S_i to sub-system S_j, n_t is a Gaussian i.i.d. process with zero mean and known variance that represents the channel noise in transmission of a component of the control or measurement vector from the ith sub-system to the jth sub-system and $\tilde{z}_t^{(ij)}$ is the corresponding channel output. $p^{(i)} \geq 0$ is the transmitter power (power of the ith transmitter antenna). Channel noises involved in transmission of different components of a control vector or measurement vector are independent of each other. Also, channel noises involved in transmission of components of different control and measurement vectors are independent of each other. Furthermore, channel noises are independent of system noises $w_t^{(i)}$s, measurement noises $v_t^{(i)}$s and initial states $x_0^{(i)}$s.

To avoid collision in exchanging information between sub-systems, a Time Division Multiple Access (TDMA) scheme [16] is used by allocating specific time slots to sub-systems to broadcast their control and measurement vectors in allocated time slot without collision.

Throughout, it is assumed that the attenuation factors $\alpha^{(ij)}$s are known, but the powers of transmitter antennas $p^{(i)}$s must be determined properly so that the received signal is the transmitted signal plus additive white Gaussian noise, as it will be explained shortly.

The controller of sub-system S_i has access to its control and measurement, i.e., $\{u_t^{(i)}, y_t^{(i)}\}$ and also a noisy version of control signals and measurements of other sub-systems communicated by AWGN channels. That is, the information pattern of each sub-system is the following:

$$I^{(i)} = \{u_t^{(i)}, y_t^{(i)}, \tilde{u}_t^{(j)}, \tilde{y}_t^{(j)}\}_{j=1, j\neq i}^{n}, \tag{3}$$

where $\tilde{u}_t^{(j)}$ and $\tilde{y}_t^{(j)}$ are the noisy version of the jth control signal $u_t^{(j)}$ and the jth measurement $y_t^{(j)}$, respectively, communicated via AWGN channels from sub-system S_j to sub-system S_i.

Cost functional and objectives: The objective of this paper is to obtain control signals $u_t^{(i)}$, $i = \{1, 2, ..., n\}$, that are a sub-optimal solution of the following optimal control problem (4), which also results in bounded mean square stability as defined in the following.

$$\min_{(u^{(1)}, u^{(2)}, ..., u^{(n)}), \text{subject to (1) and } I^{(i)}s} J$$

$$J = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathcal{E}[||X_t||_Q^2 + ||U_t||_R^2],\ \ ||X_t||_Q^2 \doteq X_t' Q X_t,\ \ ||U_t||_R^2 \doteq U_t' R U_t,$$

$$X_t \doteq (x_t^{(1)'} \quad x_t^{(2)'} \quad . \quad . \quad . \quad x_t^{(n)'})', \ U_t \doteq (u_t^{(1)'} \quad u_t^{(2)'} \quad . \quad . \quad . \quad u_t^{(n)'})',$$
$$Q = Q' \geq 0, \ R = R' > 0. \tag{4}$$

Definition 2.1: (Bounded Mean Square Stability): Consider the discrete time linear Gaussian system (1). This system is bounded mean square stabilizable if there exists a non-negative scalar M, which is a function of channel noises variances, such that the following property holds:

$$\mathcal{E}||X_t||^2 \leq M < \infty, \quad \forall t \in \mathbb{N}_+ \doteq \{0, 1, 2, ...\}.$$

III. Sub-optimal Control Technique

In this section, a sub-optimal solution for the optimal control problem (4) is presented.

Let $m_t^{(ji)} \in \mathbb{R}^{l_j}$ be a vector containing the channel noises involved in transmission of the components of the measurement vector $y_t^{(j)}$ from sub-system S_j to sub-system S_i (i.e., $\tilde{y}_t^{(j)} = \alpha^{(ji)} y_t^{(j)} + m_t^{(ji)}$). Then, considering the measurements available for each sub-system, each sub-system S_i has the following representation:

$$S_i : \begin{cases} X_{t+1} = AX_t + BU_t + W_t, \\ Y_t^{(i)} = C^{(i)} X_t + V_t^{(i)}. \end{cases}$$

$$A = \begin{pmatrix} A_{11} & A_{12} & . & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & . & A_{2n} \\ . \\ . \\ . \\ A_{n1} & A_{n2} & . & . & . & A_{nn} \end{pmatrix}, \ B = \begin{pmatrix} B_{11} & B_{12} & . & . & . & B_{1n} \\ B_{21} & B_{22} & . & . & . & B_{2n} \\ . \\ . \\ . \\ B_{n1} & B_{n2} & . & . & . & B_{nn} \end{pmatrix},$$

$$C^{(i)} = \begin{pmatrix} \alpha^{(1i)} C_1 & 0 & . & . & . & 0 & . & . & . & 0 \\ 0 & \alpha^{(2i)} C_2 & . & . & . & 0 & . & . & . & 0 \\ . \\ . \\ . \\ 0 & 0 & . & . & . & C_i & . & . & . & 0 \\ . \\ . \\ . \\ 0 & 0 & . & . & . & 0 & . & . & . & \alpha^{(ni)} C_n \end{pmatrix},$$

$$W_t \doteq (w_t^{(1)'} \quad w_t^{(2)'} \quad . \quad . \quad . \quad w_t^{(n)'})',$$

$$Y_t^{(i)} \doteq (\tilde{y}_t^{(1)'} \quad \tilde{y}_t^{(2)'} \quad . \quad . \quad . \quad y_t^{(i)'} \quad . \quad . \quad . \quad \tilde{y}_t^{(n)'})',$$

$$V_t^{(i)} \doteq (v_t^{(1)'} \quad v_t^{(2)'} \quad . \quad . \quad . \quad v_t^{(n)'})' + (m_t^{(1i)'} \quad m_t^{(2i)'} \quad . \quad . \quad . \quad 0 \quad . \quad . \quad . \quad m_t^{(ni)'})' \tag{5}$$

Now, from the standard results of Linear Quadratic Gaussian (LQG) [17] it follows that the optimal solution of the problem (4) subject to the dynamic system (5) and under the assumptions that the pair (A, B) is controllable, $(C^{(i)}, A)$ is detectable and $(A, \tilde{Q}^{\frac{1}{2}})$ is stabilizable, where $\tilde{Q} \doteq cov(W_t)$, is given as follows:

$$\begin{aligned}
U_t &= F\hat{X}_t^{(i)}, \\
F &= -(R + B'P_\infty B)^{-1}B'P_\infty A, \quad P_\infty = P_\infty' \geq 0 \\
P_\infty &= A'P_\infty A - A'P_\infty B(R + B'P_\infty B)^{-1}B'P_\infty A + \tilde{Q}. \\
\hat{X}_{t+1}^{(i)} &= A\hat{X}_t^{(i)} + BU_t + AK_t^{(i)}(Y_t^{(i)} - C^{(i)}\hat{X}_t^{(i)}), \; \hat{X}_0^{(i)} = \bar{X}_0 \doteq (\bar{x}_0^{(1)'} \quad \bar{x}_0^{(2)'} \quad . \quad . \quad . \quad \bar{x}_0^{(n)'})' \\
K_t^{(i)} &= \tilde{P}_t^{(i)}C^{(i)'}(C^{(i)'}\tilde{P}_t^{(i)}C^{(i)'} + \tilde{R}^{(i)})^{-1}, \; \tilde{R}^{(i)} \doteq cov(V_t^{(i)}) \\
\tilde{P}_{t+1}^{(i)} &= A\tilde{P}_t^{(i)}A' - A\tilde{P}_t^{(i)}C^{(i)'}(C^{(i)}\tilde{P}_t^{(i)}C^{(i)'} + \tilde{R}^{(i)})^{-1}C^{(i)}\tilde{P}_t^{(i)}A' + \tilde{Q} \\
\tilde{P}_0^{(i)} &= \bar{V}_0 \doteq cov(X_0) = diag(\bar{v}_0^{(1)} \; \bar{v}_0^{(2)} \; ... \; \bar{v}_0^{(n)}).
\end{aligned} \tag{6}$$

Now, considering the information available at sub-system S_i, the control signal $u_t^{(i)}$ is chosen as follows:

$$\begin{aligned}
u_t^{(i)} &= F_i\hat{X}_t^{(i)}, \quad (F = \begin{pmatrix} F_1 \\ F_2 \\ . \\ . \\ . \\ F_n \end{pmatrix}) \\
\hat{X}_{t+1}^{(i)} &= A\hat{X}_t^{(i)} + B_1\tilde{u}_t^{(1)} + ... + B_i u_t^{(i)} + ... + B_n\tilde{u}_t^{(n)} + AK_t^{(i)}(Y_t^{(i)} - C^{(i)}\hat{X}_t^{(i)}), \\
&\quad (B = (B_1 \quad B_2 \quad ... \quad B_n)).
\end{aligned} \tag{7}$$

Note that $F_i \in \mathbb{R}^{m_i \times (n_1+n_2+...+n_n)}$ is a block of matrix F that corresponds to $u_t^{(i)}$. Also, $B_i \in \mathbb{R}^{(n_1+n_2+...+n_n) \times m_i}$ is a block of matrix B that corresponds to $u_t^{(i)}$.

Throughout, the optimal solution to the problem (4) is referred to the solution that corresponds to the following information pattern: $I^{(i)} = \{u_t^{(i)}, y_t^{(i)}, u_t^{(j)}, y_t^{(j)}\}_{j=1, j\neq i}^n$. That is, the optimal solution corresponds to the case when the channel noises are zero and the attenuation factors are all one (i.e., no communication imperfections). For this case, the lumped system has the following representation:

$$S : \begin{cases} X_{t+1} = AX_t + BU_t + W_t, \\ Y_t = CX_t + V_t. \end{cases}$$

$$C \doteq \begin{pmatrix} C_1 & 0 & . & . & . & 0 & . & . & . & 0 \\ 0 & C_2 & . & . & . & 0 & . & . & . & 0 \\ . \\ . \\ . \\ 0 & 0 & . & . & . & C_i & . & . & . & 0 \\ . \\ . \\ . \\ 0 & 0 & . & . & . & 0 & . & . & . & C_n \end{pmatrix},$$

$$Y_t \doteq (\, y_t^{(1)'} \quad y_t^{(2)'} \quad . \quad . \quad . \quad y_t^{(i)'} \quad . \quad . \quad . \quad y_t^{(n)'} \,)',$$

$$V_t \doteq (\, v_t^{(1)'} \quad v_t^{(2)'} \quad . \quad . \quad . \quad v_t^{(n)'} \,)'. \tag{8}$$

Now, from the standard results of LQG [17], it follows that under the assumption that the pair (A, B) is controllable, (C, A) is detectable and $(A, \tilde{Q}^{\frac{1}{2}})$ is stabilizable, where $\tilde{Q} = cov(W_t)$, the optimal solution is given by (6) provided $\tilde{R}^{(i)}$ in (6) is replaced by $\tilde{R} \doteq cov(V_t)$, $C^{(i)}$ by C and $Y_t^{(i)}$ by Y_t.

Remark 3.1: The proposed control technique (7) is a sub-optimal technique because when the variances of channel noises converge to zero and also attenuation factors to one, $u_t^{(i)}$s given in (7), converge to the optimal solution, as defined above.

The communication channel between each two sub-system is an AWGN channel, as is shown in Fig. 1. This channel is subject to the limited power of transmitter antenna ($P < \infty$). That is, when the power of transmitted signal (i.e., $\mathcal{E}[X^2]$) is less than the power of transmitter antenna ($\mathcal{E}[X^2] \leq P$), then $Y = X + Z$, where Y is the output of the channel and Z is an i.i.d. Gaussian channel noise [18]. When this constraint is violated (i.e., $\mathcal{E}[X^2] > P$), then $Y = \bar{X} + Z$, where $\bar{X}$ is a distorted version of transmitted signal (i.e., $\bar{X}$ is not equivalent to X and may be very different from X). In the set up considered in this paper, there is an AWGN channel from sub-system i to sub-system j, in which via this channel, each component of the measurement vector $y_t^{(i)}$ and each component of the control vector $u_t^{(i)}$ must be transmitted from sub-system i to sub-system j. Hence, to have a transmission so that the received signal is the transmitted signal

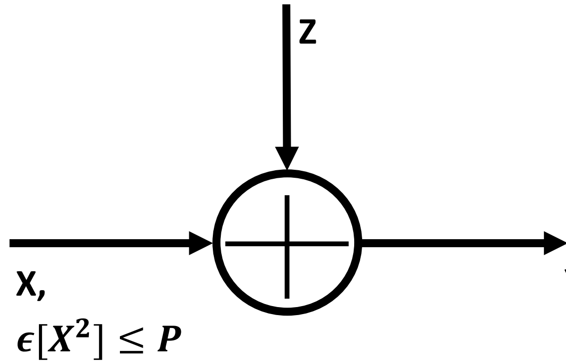

Fig. 1. AWGN channel.

plus additive white Gaussian noise, the power of transmitter antenna must be high enough.

Remark 3.2: Following the above discussion, let $y_t^{(ih)}$ and $u_t^{(io)}$ be the hth component of the measurement vector $y_t^{(i)}$ and the oth component of the control vector $u_t^{(i)}$, respectively. Then, to have a transmission so that the received signal is the transmitted signal plus additive white Gaussian noise, the power of the ith transmitter antenna is chosen such that the following inequality holds [18]:

$$p^{(i)} \geq \sup_{t \in \mathbb{N}_+ \doteq \{0,1,2,...\}} \{\max_h \mathcal{E}[y_t^{(ih)}]^2, \max_o \mathcal{E}[u_t^{(io)}]^2\}, \; i \in \{1, 2, ..., n\}. \tag{9}$$

Remark 3.3: The power of transmitted signal must be less than the power of transmitter antenna to have the above transmission. Hence, We choose a suitable transmitter with proper power. Nevertheless, as the transmitter power is limited ($p^{(i)} < \infty$), transmission is subject to channel noise due to the limited transmitter power. This channel noise results in non-classical and different information patterns; and hence, the channel noise and therefore the power constraint ($p^{(i)} < \infty$) that results in channel noise are critical for the problem considered in this paper, as we need to use non-classical design methodologies to find solution to the problem (4). Note that when the power of transmitter is unlimited, the transmitted signal can be amplified so high that the channel noise becomes negligible compared to transmitted signal.

IV. Stability Results

In this section, it is shown that under some conditions the proposed sub-optimal control technique results in bounded mean square stability as defined in Definition 2.1. Then, the power to be allocated to each transmitter antenna is calculated so that the received signal is the transmitted signal plus additive white Gaussian noise.

A. Stability result

The stability of the proposed technique is shown in the following proposition.

Proposition 4.1: Consider the distributed dynamic system (1) over AWGN communication network (2). Suppose that the control signals are given by (7), the powers of transmitter antennas satisfy the inequality (9) and the pairs (A, B) is controllable, $(C^{(i)}, A)$, $i \in \{1, 2, ..., n\}$ are detectable and $(A, \tilde{Q}^{\frac{1}{2}})$ is stabilizable. Then, the system (1) is bounded mean square stable if the following matrix $\mathcal{A}_c$ is a stable matrix.

$$\mathcal{A}_c = \begin{pmatrix} A+BF & -B_1F_1 & -B_2F_2 & . & . & . & -B_{n-1}F_{n-1} - B_nF_n & -B_nF_n \\ \beta_{21} & \beta_{22} & \beta_{23} & . & . & . & \beta_{2\ n-1} & \beta_{2n} \\ . & & & & & & & \\ . & & & & & & & \\ . & & & & & & & \\ \beta_{n-1\ 1} & \beta_{n-1\ 2} & \beta_{n-1\ 3} & . & . & . & \beta_{n-1\ n-1} & \beta_{n-1\ n} \\ \beta_{n1} & \beta_{n2} & \beta_{n3} & . & . & . & \beta_{n\ n-1} & \beta_{nn} \end{pmatrix},$$

$$\begin{aligned}
&\beta_{21} = \sum_{j=1, j\neq 1}^{n} B_jF_j(1-\alpha^{(j1)}),\ \beta_{22} = A - AK_\infty^{(1)}C^{(1)},\ \beta_{23} = -B_2F_2(1-\alpha^{(21)}),\\
&\beta_{2\ n-1} = B_{n-1}F_{n-1}\alpha^{(n-1\ 1)} - B_nF_n(1-\alpha^{(n1)}),\ \beta_{2n} = -B_nF_n(1-\alpha^{(n1)}),\\
&\beta_{n-1\ 1} = \sum_{j=1, j\neq n-1}^{n} B_jF_j(1-\alpha^{(j\ n-1)}),\ \beta_{n-1\ 2} = -B_1F_1(1-\alpha^{(1\ n-1)}),\\
&\beta_{n-1\ 3} = -B_2F_2(1-\alpha^{(2\ n-1)}),\ \beta_{n-1\ n-1} = A - AK_\infty^{(n-1)}C^{(n-1)} - B_nF_n(1-\alpha^{(n\ n-1)}),\\
&\beta_{n-1\ n} = -B_nF_n(1-\alpha^{(n\ n-1)}),\ \beta_{n1} = \sum_{j=1}^{n-2} B_jF_j(\alpha^{(j\ n-1)} - \alpha^{(jn)})\\
&+B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)}) - B_nF_n(1-\alpha^{(n\ n-1)})\\
&\beta_{n2} = B_1F_1\alpha^{(1n)} - B_1F_1\alpha^{(1\ n-1)},\ \beta_{n3} = B_2F_2\alpha^{(2n)} - B_2F_2\alpha^{(2\ n-1)},\\
&\beta_{n\ n-1} = AK_\infty^{(n-1)}C^{(n-1)} - AK_\infty^{(n)}C^{(n)} - B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)})\\
&+B_nF_n(1-\alpha^{(n\ n-1)}),\\
&\beta_{nn} = A - AK_\infty^{(n)}C^{(n)} - B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)}) + B_nF_n(1-\alpha^{(n\ n-1)}),
\end{aligned} \tag{10}$$

where $K_\infty^{(i)} \doteq \lim_{t\to\infty} K_t^{(i)}$ is the steady state Kalman filter gain.

Proof: Let $f_t^{(ji)} \in \mathbb{R}^{m_j}$ with covariance matrix $\Sigma_f^{(ji)}$ is a vector containing the channel noises involved in transmission of components of the vector $u_t^{(j)}$ from sub-system S_j to sub-system S_i. Define the vector L_t as follows:

$$L_t \doteq \begin{pmatrix} X_t \\ E_t^{(1)} \\ . \\ . \\ . \\ E_t^{(n-1)} \\ E_t^{(n-1\ n)} \end{pmatrix}, \quad E_t^{(n-1\ n)} \doteq \hat{X}_t^{(n-1)} - \hat{X}_t^{(n)}.$$

Then, from the recursive equations for X_t and $\hat{X}_t^{(i)}$, $i = \{1, 2, ..., n\}$ given in (5) and (7), we have the following dynamic system for L_t:

$$L_{t+1} = \mathcal{G}_t L_t + \mathcal{N}_t, \tag{11}$$

$$\mathcal{G}_t = \begin{pmatrix} A+BF & -B_1F_1 & -B_2F_2 & . & . & . & -B_{n-1}F_{n-1} - B_nF_n & -B_nF_n \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & . & . & . & \gamma_{2\ n-1} & \gamma_{2n} \\ . \\ . \\ . \\ \gamma_{n-1\ 1} & \gamma_{n-1\ 2} & \gamma_{n-1\ 3} & . & . & . & \gamma_{n-1\ n-1} & \gamma_{n-1\ n} \\ \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & . & . & . & \gamma_{n\ n-1} & \gamma_{nn} \end{pmatrix},$$

$$\gamma_{21} = \sum_{j=1, j\neq 1}^{n} B_jF_j(1-\alpha^{(j1)}),\ \gamma_{22} = A - AK_t^{(1)}C^{(1)},\ \gamma_{23} = -B_2F_2(1-\alpha^{(21)}),$$

$$\gamma_{2\ n-1} = B_{n-1}F_{n-1}\alpha^{(n-1\ 1)} - B_nF_n(1-\alpha^{(n1)}),\ \gamma_{2n} = -B_nF_n(1-\alpha^{(n1)}),$$

$$\gamma_{n-1\ 1} = \sum_{j=1, j\neq n-1}^{n} B_jF_j(1-\alpha^{(j\ n-1)}),\ \gamma_{n-1\ 2} = -B_1F_1(1-\alpha^{(1\ n-1)}),$$

$$\gamma_{n-1\ 3} = -B_2F_2(1-\alpha^{(2\ n-1)}),\ \gamma_{n-1\ n-1} = A - AK_t^{(n-1)}C^{(n-1)} - B_nF_n(1-\alpha^{(n\ n-1)}),$$

$$\gamma_{n-1\ n} = -B_nF_n(1-\alpha^{(n\ n-1)}),\ \gamma_{n1} = \sum_{j=1}^{n-2} B_jF_j(\alpha^{(j\ n-1)} - \alpha^{(jn)})$$

$$+B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)}) - B_nF_n(1-\alpha^{(n\ n-1)})$$

$$\gamma_{n2} = B_1F_1\alpha^{(1n)} - B_1F_1\alpha^{(1\ n-1)},\ \gamma_{n3} = B_2F_2\alpha^{(2n)} - B_2F_2\alpha^{(2\ n-1)},$$

$$\gamma_{n\ n-1} = AK_t^{(n-1)}C^{(n-1)} - AK_t^{(n)}C^{(n)} - B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)})$$

$$+B_nF_n(1-\alpha^{(n\ n-1)}),$$

$$\gamma_{nn} = A - AK_t^{(n)}C^{(n)} - B_{n-1}F_{n-1}(1-\alpha^{(n-1\ n)}) + B_nF_n(1-\alpha^{(n\ n-1)}),$$

$$\mathcal{N}_t = \begin{pmatrix} W_t \\ -\sum_{j=1,j\neq 1}^{n} B_jF_jf_t^{(j1)} - AK_t^{(1)}C^{(1)}V_t^{(1)} + W_t \\ . \\ . \\ . \\ -\sum_{j=1,j\neq n-1}^{n} B_jF_jf_t^{(j\ n-1)} - AK_t^{(n-1)}C^{(n-1)}V_t^{(n-1)} + W_t \\ e_t \end{pmatrix},$$

$$e_t \doteq AK_t^{(n-1)}C^{(n-1)}V_t^{(n-1)} - AK_t^{(n)}C^{(n)}V_t^{(n)} + \sum_{j=1,j\neq n-1}^{n} B_jF_jf_t^{(j\ n-1)} - \sum_{j=1,j\neq n}^{n} B_jF_jf_t^{(jn)}. \tag{12}$$

The system (11) is a Gaussian system, that is, the solution of the recursive equation (11), i.e., L_t is a Gaussian random variable as L_0 and $\mathcal{N}_t$ are Gaussian random variables. Also, under the assumption that the pairs $(C^{(i)}, A)$, $i = \{1, 2, ..., n\}$ are detectable and the pair $(A, \tilde{Q}^{\frac{1}{2}})$ is stabilizable, it follows from the standard results of Kalman filtering [17] that $K_t^{(i)} \to K_\infty^{(i)}$, where $K_\infty^{(i)}$ is the steady state Kalman filter gain. Hence, under this assumption, we have $\mathcal{G}_t \to \mathcal{A}_c$. Now, by expanding the dynamic system (11) and representation of the vector L_t as a linear function of L_0 and $\{\mathcal{N}_k\}_{k=0}^{t-1}$ and then computing $\mathcal{E}[L_tL_t']$, it follows that $\lim_{t\to\infty}\mathcal{E}[L_tL_t'] < \infty$ as the matrix $\mathcal{A}_c$ is a stable matrix. Obviously, $\mathcal{E}[L_tL_t']$ is bounded at finite times. Hence, under the assumptions of the proposition, the Gaussian vector L_t; and therefore, X_t along with $E_t^{(1)}$, $E_t^{(2)}$, ..., $E_t^{(n-1\ n)}$ are bounded in mean square sense.

Remark 4.2: i) Note that the matrix $\mathcal{A}_c$ as given in (10) involves steady state Kalman filter gains $K_\infty^{(i)} = \tilde{P}_\infty^{(i)}C^{(i)\prime}(C^{(i)}\tilde{P}_\infty^{(i)}C^{(i)\prime} + R^{(i)})^{-1}$, where $\tilde{P}_\infty^{(i)}$ is the steady state solution of the forward Riccati equation given in (6) and $\tilde{R}^{(i)} = cov(V_t^{(i)})$, where $V_t^{(i)}$ as given in (5) involves measurement noises as well as channel noises $m_t^{(ji)}$. Hence, the system matrix $\mathcal{A}_c$ is also affected by the channel noises $m_t^{(ji)}$. From the way we defined the vector L_t, the effects of the channel noises $f_t^{(ji)}$ is also seen in the vector $\mathcal{N}_t$. In the presence of channel noises $f_t^{(ji)}$, the covariance matrix of the vector $\mathcal{N}_t$ is obviously bigger than the covariance matrix of this matrix without channel noises $f_t^{(ji)}$. This results in the bounded mean square stability with a bigger upper bound on $\mathcal{E}||X_t||^2$ when there are channel noises $f_t^{(ji)}$. That is, the quality of the response for this case is not as good as the quality of the response for the case with no channel noises $f_t^{(ji)}$.

ii) If the measurement and control vectors are communicated via channels with attenuation factors 1 (i.e., $\tilde{u}_t^{(i)} = u_t^{(i)} + f_t^{(ij)}$ and $\tilde{y}_t^{(i)} = y_t^{(i)} + m_t^{(ij)}$, $j \in \{1, 2, ..., n\}$, $j \neq i$), then the matrix

$\mathcal{A}_c$ is reduced as follows:

$$\mathcal{A}_c = \begin{pmatrix} A+BF & -B_1F_1 & . & . & . & -B_{n-1}F_{n-1}-B_nF_n & -B_nF_n \\ 0 & A-AK_\infty^{(1)}C^{(1)} & . & . & . & 0 & 0 \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ 0 & 0 & . & . & . & A-AK_\infty^{(n-1)}C^{(n-1)} & 0 \\ 0 & 0 & . & . & . & AK_\infty^{(n-1)}C^{(n-1)}-AK_\infty^{(n)}C^{(n)} & A-AK_\infty^{(n)}C^{(n)} \end{pmatrix}.$$

Now, if the matrix $K_\infty^{(n-1)}C^{(n-1)} = K_\infty^{(n)}C^{(n)}$, then the above matrix is reduced to an upper triangular matrix; and consequently, it is a stable matrix because following the standard results of LQG and Kalman filtering the matrices $A+BF$, $A-AK_\infty^{(1)}C^{(1)}$, ... and $A-AK_\infty^{(n)}C^{(n)}$ are stable matrices.

iii) If measurements are communicated without any imperfections (i.e., $\tilde{y}_t^{(i)} = y_t^{(i)}$), then $K_\infty^{(n-1)}C^{(n-1)}$ $= K_\infty^{(n)}C^{(n)}$. Also, when channel noises in communication of measurements are small; while attenuation factors in exchanging information between sub-systems are one, then $\rho(K_\infty^{(n-1)}C^{(n-1)} - K_\infty^{(n)}C^{(n)}) \approx 0$; and consequently, the matrix $\mathcal{A}_c$ given above has a quasi upper triangular structure and hence it is a stable matrix.

iv) By multiplying the received measurement and control vectors by the inverse of attenuation factor $\alpha^{(ji)}$ involved in transmission of measurement and control vectors from sub-system S_j to sub-system S_i, we can reach to a communication network that is equivalent to an AWGN network with attenuation factors that are all one; but with higher channel noise variance.

B. The power to be allocated to each transmitter antenna

From the inequality (9), it follows that to calculate the power of each transmitter antenna for having a transmission so that the received signal is the transmitted signal plus additive white Gaussian noise, it is enough to compute $\mathcal{E}[y_t^{(i)}y_t^{(i)'}]$ and $\mathcal{E}[u_t^{(i)}u_t^{(i)'}]$. This is done in the following.

We first notice the following equalities

$$\begin{aligned} \mathcal{E}[u_t^{(i)}u_t^{(i)'}] &= F_i\mathcal{E}[\hat{X}_t^{(i)}\hat{X}_t^{(i)'}]F_i', \\ \mathcal{E}[y_t^{(i)}y_t^{(i)'}] &= C_i\mathcal{E}[x_t^{(i)}x_t^{(i)'}]C_i' + \Sigma_v^{(i)}. \end{aligned} \tag{13}$$

Now, as $\mathcal{E}[\hat{X}_t^{(i)}\hat{X}_t^{(i)'}] = \mathcal{E}[X_tX_t'] - \mathcal{E}[E_t^{(i)}E_t^{(i)'}]$, to compute $\mathcal{E}[y_t^{(i)}y_t^{(i)'}]$ and $\mathcal{E}[u_t^{(i)}u_t^{(i)'}]$, it is enough to compute $\mathcal{E}[X_tX_t']$ and $\mathcal{E}[E_t^{(i)}E_t^{(i)'}]$, $i = \{1,2,...,n\}$. This is done in the following.

To compute $\mathcal{E}[X_tX_t']$ and $\mathcal{E}[E_t^{(i)}E_t^{(i)'}]$, $i = 1,2,...,n-1$, we use the recursive equation (11) which contains X_t, $E_t^{(1)}$, ..., $E_t^{(n-1)}$. From this recursive equation we have the following recursive equation for $\mathcal{E}[L_tL_t']$.

$$\mathcal{E}[L_{t+1}L_{t+1}'] = \mathcal{G}_t\mathcal{E}[L_tL_t']\mathcal{G}_t' + \mathcal{E}[\mathcal{N}_t\mathcal{N}_t'],$$

$$\mathcal{E}[L_0L_0'] = \begin{pmatrix} \bar{V}_0 + \bar{X}_0\bar{X}_0' & 0 & . & . & . & 0 & 0 \\ \bar{V}_0 & \bar{V}_0 & . & . & . & \bar{V}_0 & 0 \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ \bar{V}_0 & \bar{V}_0 & . & . & . & \bar{V}_0 & 0 \\ 0 & 0 & . & . & . & 0 & 0 \end{pmatrix},$$

$$\mathcal{E}[\mathcal{N}_t\mathcal{N}_t'] = \begin{pmatrix} \tilde{Q} & \tilde{Q} & . & . & . & \tilde{Q} & \tilde{Q} \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ \tilde{Q} & \tilde{Q} & . & . & . & z_{nn} & z_{n+1\ n}' \\ 0 & 0 & . & . & . & z_{n+1\ n} & z_{n+1\ n+1} \end{pmatrix},$$

$$z_{n+1\ n} = -AK_t^{(n-1)}C^{(n-1)}\tilde{R}^{(n-1)}C^{(n-1)'}K_t^{(n-1)'}A' - \sum_{j=1,j\neq n-1}^{n} B_jF_j\Sigma_f^{(j\ n-1)}F_j'B_j',$$

$$z_{n+1\ n+1} = AK_t^{(n-1)}C^{(n-1)}\tilde{R}^{(n-1)}C^{(n-1)'}K_t^{(n-1)'}A' + AK_t^{(n)}C^{(n)}\tilde{R}^{(n)}C^{(n)'}K_t^{(n)'}A'$$
$$+\sum_{j=1}^{n-1} B_jF_j\Sigma_f^{(jn)}F_j'B_j' + \sum_{j=1,j\neq n-1}^{n} B_jF_j\Sigma_f^{(j\ n-1)}F_j'B_j' - AK_t^{(n-1)}C^{(n-1)}\Sigma_VC^{(n)'}K_t^{(n)'}A'$$
$$-AK_t^{(n)}C^{(n)}\Sigma_VC^{(n-1)'}K_t^{(n-1)'}A', \quad \Sigma_V \doteq diag(\Sigma_v^{(1)}\ \Sigma_v^{(2)}\ ...\ \Sigma_v^{(n)}),$$
$$z_{hh}(h = 2, ..., n) = \sum_{j=1,j\neq h-1}^{n} B_jF_j\Sigma_f^{(j\ h-1)}F_j'B_j' + AK_t^{(h-1)}C^{(h-1)}\tilde{R}^{(h-1)}C^{(h-1)'}K_t^{(h-1)'}A'$$
$$+\tilde{Q}. \tag{14}$$

Now, from the recursive equation (14), $\mathcal{E}[L_tL_t']$ and hence $\mathcal{E}[X_tX_t']$ (and therefore $\mathcal{E}[x_t^{(i)}x_t^{(i)'}]$, $i = \{1, 2, ..., n\}$) and $\mathcal{E}[E_t^{(i)}E_t^{(i)'}]$, $i = \{1, 2, ..., n-1\}$, are computed.

To compute $\mathcal{E}[E_t^{(n)}E_t^{(n)'}]$, we notice that $E_t^{(n)} = E_t^{(n-1\ n)} + E_t^{(n-1)}$; and therefore,

$$\begin{aligned} \mathcal{E}[E_t^{(n)}E_t^{(n)'}] &= \mathcal{E}[E_t^{(n-1\ n)}E_t^{(n-1\ n)'}] + \mathcal{E}[E_t^{(n-1\ n)}E_t^{(n-1)'}] \\ &\quad +\mathcal{E}[E_t^{(n-1)}E_t^{(n-1\ n)'}] + \mathcal{E}[E_t^{(n-1)}E_t^{(n-1)'}], \end{aligned}$$

where $\mathcal{E}[E_t^{(n-1\ n)}E_t^{(n-1\ n)'}]$, $\mathcal{E}[E_t^{(n-1\ n)}E_t^{(n-1)'}]$,$\mathcal{E}[E_t^{(n-1)}E_t^{(n-1\ n)'}]$ and $\mathcal{E}[E_t^{(n-1)}E_t^{(n-1)'}]$ are computed from the recursive equation (14). Consequently, using this recursive equation, the equal-

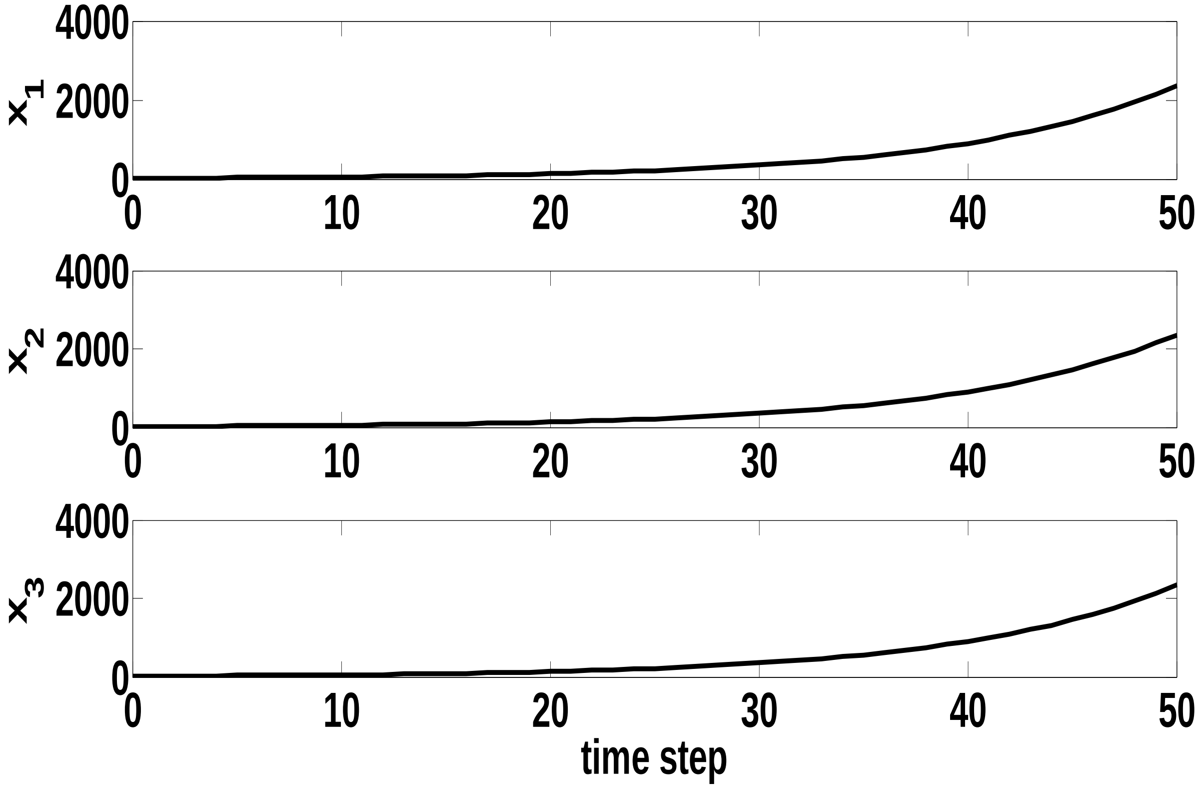

Fig. 2. State trajectories without control inputs.

ity $\mathcal{E}[\hat{X}_t^{(i)}\hat{X}_t^{(i)'}] = \mathcal{E}[E_t^{(i)}E_t^{(i)'}] - E[X_tX_t']$ and the equalities (13), $\mathcal{E}[u_t^{(i)}u_t^{(i)'}]$ and $\mathcal{E}[y_t^{(i)}y_t^{(i)'}]$, $i = \{1, 2, ..., n\}$, are obtained. Hence, the powers to be allocated to transmitter antennas are calculated from (9) to have a transmission so that the received signal is the transmitted signal plus additive white Gaussian noise.

V. Simulation Results

In this section, the satisfactory performance of the proposed stabilizing sub-optimal control technique for bounded mean square stability of the distributed dynamic system (1) is illustrated using computer simulations.

For simulation study, we are concerned in this section with a distributed system with three

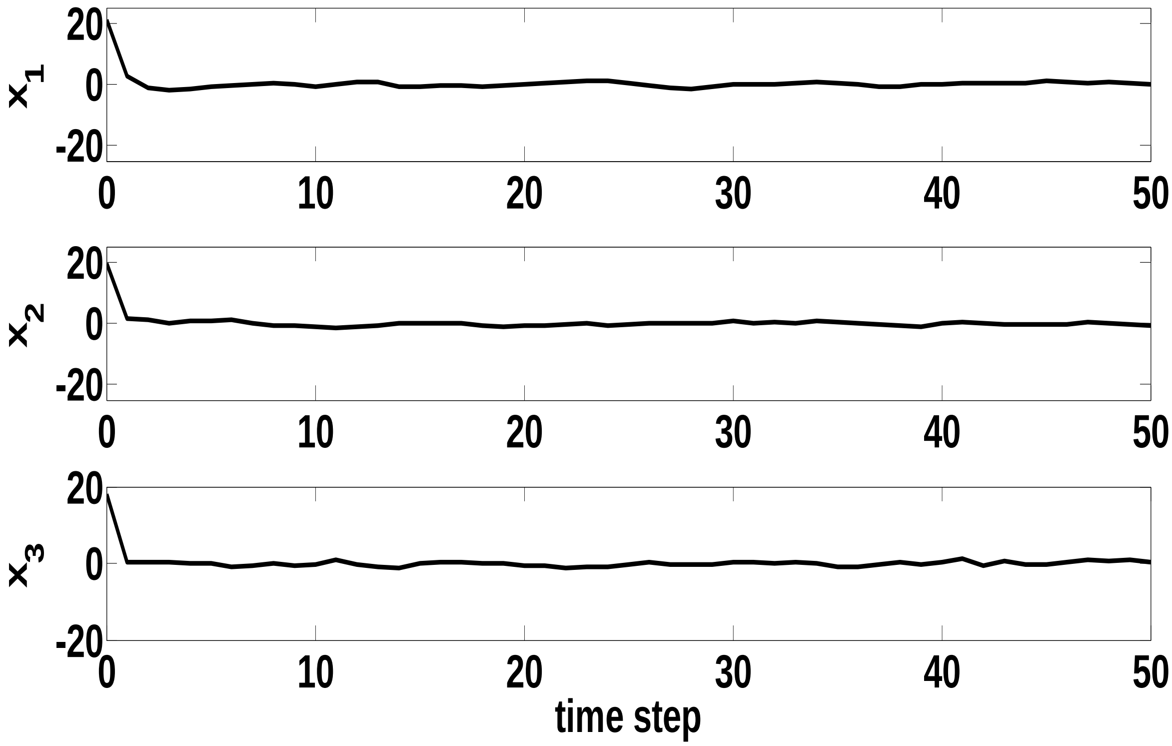

Fig. 3. State trajectories of the controlled system when communication between sub-systems are perfect.

scalar interacting sub-systems of the form (1) with augmented matrices A and B given as follows:

$$A = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Variances of system noise and measurement noise of each sub-system are assumed to be 0.1. The initial state of each sub-system is a random variable with mean 20 and variance 1. System noise, measurement noise and initial state of different sub-systems are independent of each other. System noise, measurement noise and initial state of each sub-system are also independent of each other. Weighting matrices Q and R are set to be $10I_3$ and I_3, respectively. $C_1 = C_2 = C_3 = 1$. The attenuation factors are set to be one.

Fig. 2 illustrates the response of this system without control inputs. As is clear from Fig. 2, without control inputs, the distributed system is unstable. This result is expected as the matrix A is an unstable matrix.

To stabilize the system, we use the proposed control technique. For the ideal case of no channel noise in exchanging measurements and control signals between sub-systems, the proposed control technique is the optimal technique. Fig. 3 illustrates the response of the controlled system for this case. As is clear from this figure and as expected from Proposition 4.1, the proposed technique stabilizes the system around the origin in bounded mean square sense.

Now, suppose communication between sub-systems is noisy and subject to channel noises with the variances of 0.1 in exchanging measurements and 1 in exchanging control signals. Here, it is assumed that the attenuation factors are 1. For this case, $\max_{t\in\mathbb{N}_+}\mathcal{E}[y_t^{(1)}]^2 = 1.86$, $\max_{t\in\mathbb{N}_+}\mathcal{E}[y_t^{(2)}]^2 = 1.86$ and $\max_{t\in\mathbb{N}_+}\mathcal{E}[y_t^{(3)}]^2 = 1.86$. Also, $\max_{t\in\mathbb{N}_+}\mathcal{E}[u_t^{(1)}]^2 = \max_{t\in\mathbb{N}_+}\mathcal{E}[u_t^{(2)}]^2$ $= \max_{t\in\mathbb{N}_+}\mathcal{E}[u_t^{(3)}]^2 = 2.14$; and hence, we choose the powers of transmitter antennas as follows: $p^{(1)} = 2.14$, $p^{(2)} = 2.14$ and $p^{(3)} = 2.14$ to have a transmission so that the received signal is the transmitted signal plus additive white Gaussian noise. Fig.4 illustrates the response of the system for this case. As is clear from Fig. 4 the performance of the proposed control technique for the conditions simulated is close to the optimal performance.

Similarly, in [12] a bounded mean square stabilizing technique was proposed for a linear Gaussian system with distributed sub-systems when only measurement vectors are exchanged between distributed sub-systems. In [12] it is assumed that control vectors are exchanged without any imperfections. Fig. 5 illustrates the response of the controlled technique proposed in [12] applied to the system considered in this section when the variances of the channel noise in exchanging measurements and control signals are 0,1 and 1, respectively. The attenuation factors are assumed to be 1. For this case $p^{(1)} = 3.56$, $p^{(2)} = 3.56$ and $p^{(3)} = 3.56$. As is clear from this figure, although the transmitter powers for this case are more than those of the previous simulation, the performance for this case is not as good as the previous simulation.

Fig. 6 illustrates the response of our proposed technique when the variances of the channel noise in exchanging measurements and control signals are 0.1 and 10, respectively. Fig. 7 illustrates the response of the technique of [12] for the same conditions. As is clear from these figures for the cases with large noise in exchanging control signal, the performance of our technique is much better.

VI. Conclusion and Direction for Future Research

In this paper, a sub-optimal control technique was proposed for a linear Gaussian system with a few distributed interacting sub-systems. Controller of each sub-system has only access to its own measurement and a noisy version of measurement and control vectors of other sub-systems

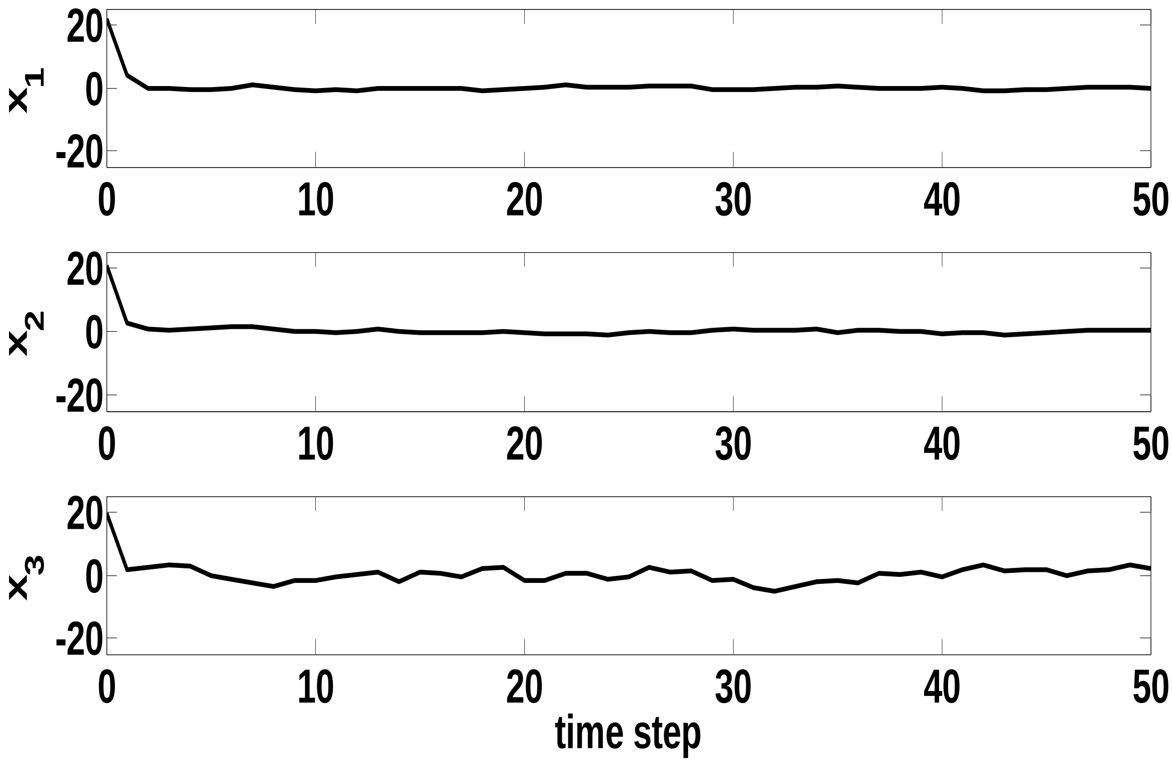

Fig. 4. State trajectories of the controlled system when variances of channel noise in exchanging measurements and controls are 0.1 and 1, respectively, and attenuation factors are 1.

that are communicated by an AWGN communication network. The power to be allocated to each transmitter antenna was calculated so that the received signal is the transmitted signal plus additive white Gaussian noise. Under some conditions, it was shown that the proposed sub-optimal control technique results in bounded mean square stability. The satisfactory performance of the proposed technique in stabilizing a distributed dynamic system was also illustrated by computer simulations.

For future, it is interesting to extend these results to large scale distributed systems, which have large number of distributed sub-systems, by presenting sub-optimal control techniques with the following property that each controller has only access to noisy version of measurements and control signals of neighboring sub-systems. It is also interesting to equip the proposed technique with proper coding technique to compensate the effects of channel noise. These problems are

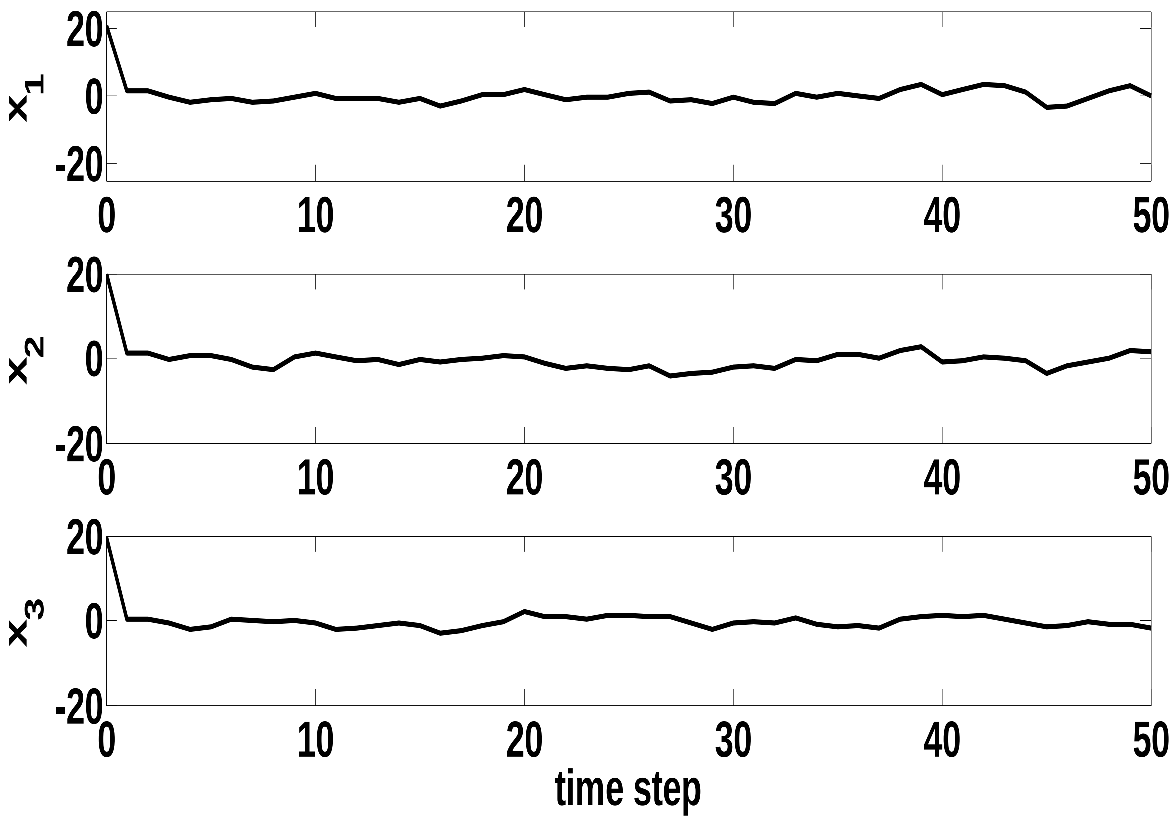

Fig. 5. State trajectories of the controlled system of [12] when variances of channel noise in exchanging measurements and controls are 0.1 and 1, respectively, and attenuation factors are 1.

left for future investigation.

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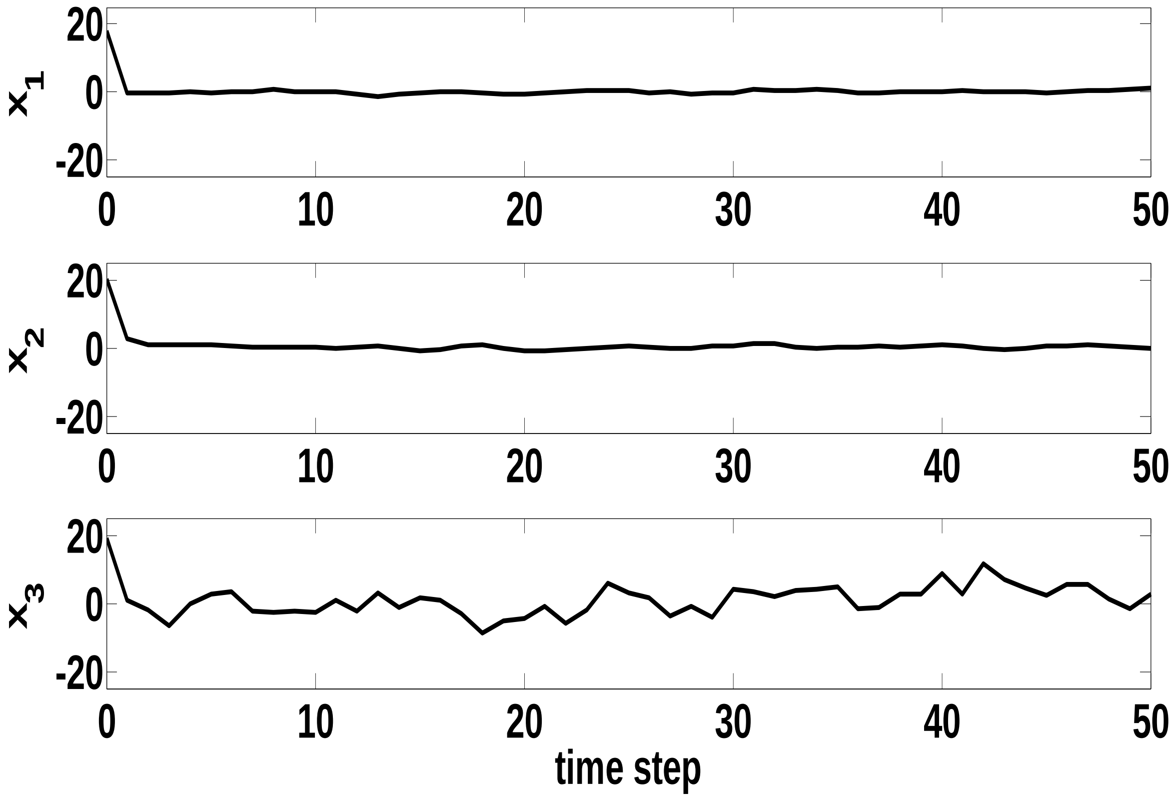

Fig. 6. State trajectories of the controlled system when variances of channel noise in exchanging measurements and controls are 0.1 and 10, respectively, and attenuation factors are 1.

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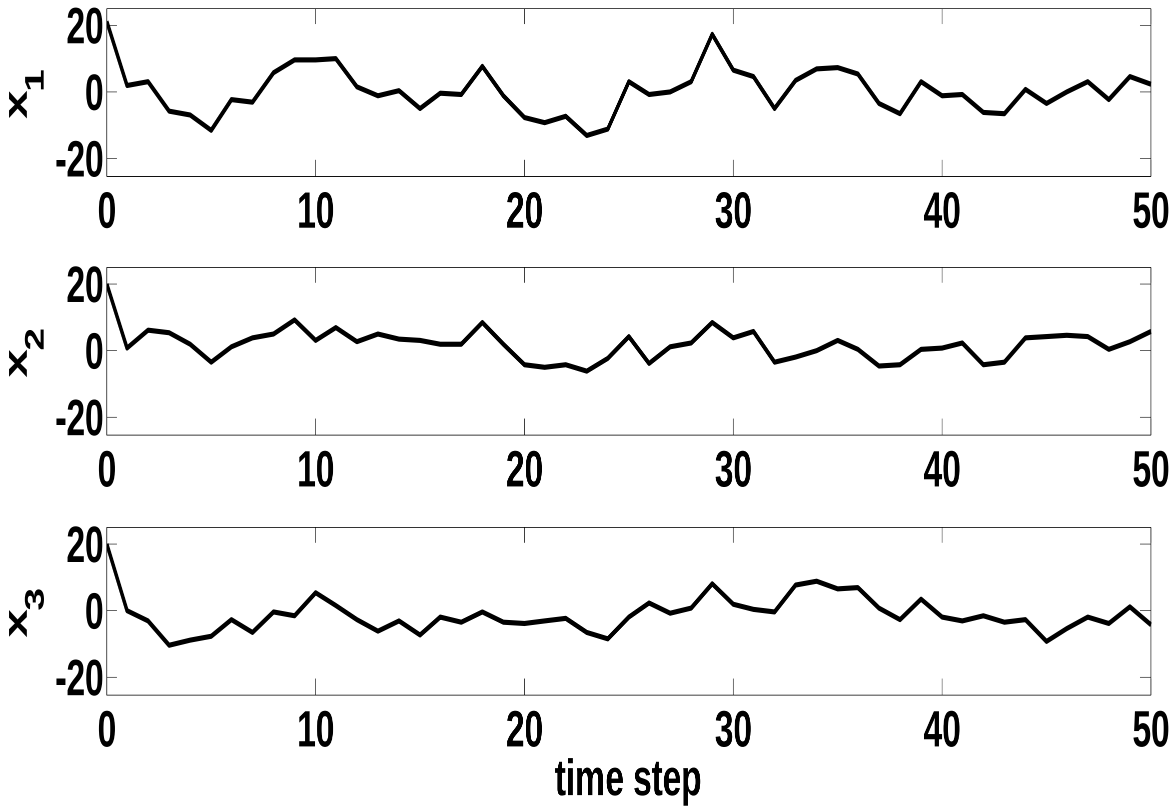

Fig. 7. State trajectories of the controlled system of [12] when variances of channel noise in exchanging measurements and controls are 0.1 and 10, respectively, and attenuation factors are 1.

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