Measurement and Control of Nonlinear Dynamic Systems over the Internet (IoT): Applications in Remote Control of Autonomous Vehicles ⋆

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Abstract

This paper presents a new technique for almost sure asymptotic state tracking, stability and reference tracking of nonlinear dynamic systems by remote controller over the packet erasure channel, which is an abstract model for transmission via WiFi and the Internet. By implementing a suitable linearization method, a proper encoder and decoder are presented for tracking the state trajectory of nonlinear systems at the end of communication link when the measurements are sent through the packet erasure channel. Then, a controller for reference tracking of the system is designed. In the proposed technique linearization is applied when the error between the states and an estimate of these states at the decoder increases. It is shown that the proposed technique results in almost sure asymptotic reference tracking (and hence stability) over the packet erasure channel. The satisfactory performance of the proposed state trajectory and reference tracking technique is illustrated by computer simulations by applying this technique on the unicycle model, which represents the dynamic of autonomous vehicles.

Key words: Networked control systems; Internet of Things (IoT); nonlinear dynamic system; autonomous vehicles; the unicycle model.

1 Introduction

1.1 Motivations and Background

In recent years, extensive research activity has been devoted to measurement and control over communication links subject to imperfections, e.g., packet dropout, distortion due to limited bandwidth, etc. Questions of this kind are motivated by future generation of mobile communications, such as 5G and tactile Internet that are explicitly intended to meet latency requirements for control applications [1],[2]. Real - time reliable data reconstruction at the end of communication links (state tracking) and stability over communication channels subject to imperfections have been an active research direction in recent years [3] - [31]. Fig. 1 illustrates a basic block diagram for studying the question of real - time reliable data reconstruction and stability subject to communication imperfections. The block diagram of Fig. 1 can correspond to real applications, such as the tactile control of small autonomous vehicles (e.g. miniature drones). As these vehicles are supplied with limited capacity onboard batteries, the data from the vehicle to the remote controller (located at the control room) must be transmitted with minimum possible power; and therefore, this data transfer is subject to imperfections (e,g., packet dropout, distortion, etc.). While, the data from the control room to the vehicle can be transmitted with high power; and therefore, the transmission of data from remote controller to the vehicle can be considered almost without errors and limitations, as is shown in the block diagram of Fig. 1.

Various publications have introduced necessary and sufficient conditions for reliable data reconstruction and stability of Fig. 1, e.g., [3]-[5], [9], [10]. In most of these references, these conditions are given in the form of a lower bound on the capacity in terms of the rate of change of dynamic system (measured in bits per time step). In particular, it is already known that the eigenvalues rate condition (i.e. $C \geq \sum_{\{i;\lambda_i(A)\geq 1\}} \log|\lambda_i(A)|$ where C is the Shannon capacity and $\lambda_i(A)$s are the eigenvalues of the system matrix A of the linear dis-

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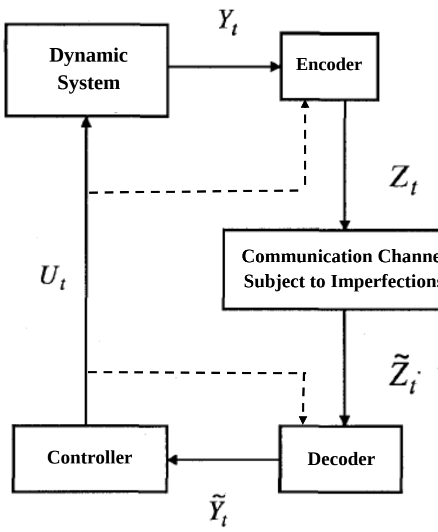

Fig. 1. A dynamic system over a communication channel subject to imperfections.

crete time-invariant system) represents the minimum capacity under which there are encoding and stabilizing schemes for reliable data reconstruction and stability of linear time-invariant systems [3]-[7]. In this paper in addition of reliable data reconstruction and stability, we concerned with reference tracking. In [11] and [12], the authors studied the optimal tracking performance of SISO linear time-invariant systems over communication channels subject to imperfections, e.g., packet dropout, induced delays. The tracking performance considered in [11] and [12] is measured by the energy of the error signal between the plant output and the reference signal. They present the explicit expressions for the minimal tracking error with or without communication constraints and show that the tracking performance depends on the non-minimum phase zeros, unstable poles and the parameters describing communication channel.

As most of important applications of networked control systems involve nonlinear systems, we study nonlinear networked control systems in this paper. In [9], the authors considered the stability of a fully observed noiseless nonlinear time-invariant dynamic systems subject to unknown initial condition over the limited capacity digital noiseless channel. This problem can be modeled by the basic block diagram of Fig. 1 described by such dynamic systems and the digital noiseless channel. In [9], the authors developed the notion of topological feedback entropy rate for completely deterministic system that measures the fastest rate at which the initial state information can be generated. They found a necessary and sufficient condition on the channel capacity for stability over the digital noiseless channel. In [10] the authors considered the global asymptotic stability of a fully observed continuous time-invariant nonlinear dynamic systems where the measurements must be received by controller at discrete times; and the data available to the controller is a stream of binaries. That is, they considered the control/communication system of Fig. 1 described by such nonlinear dynamic system over the digital noiseless channel; and they found a sufficient condition for stability relating the channel capacity to parameters describing the nonlinear dynamic system. [13] is concerned with tracking a vector of signal process generated by a family of distributed (geographically separated) nonlinear noisy dynamic subsystems over the packet erasure channel. Nonlinear subsystems are subject to bounded external disturbances. Measurements are also subject to bounded noises. For this system and channel, subject to constraints on transmission rates, cross over probabilities and the Lipschitz constants, a simple technique is presented ensuring tracking state trajectory with bounded mean absolute error. In [14], it is shown that the desired estimation of a nonlinear systems with limited information is impossible for bit rates which are lower than the so-called estimation entropy. Furthermore, it is proved that the derived upper bound on the estimation entropy matches the average bit rate that guarantees the desired estimation. In [15], the authors presented a necessary condition for mean square exponential reliable data reconstruction of noiseless nonlinear dynamic systems over the real erasure channel in terms of erasure probability and positive Lyapunov exponents. In [16], the authors also presented a necessary condition in terms of the positive Lyapunov exponents for the stability of nonlinear noiseless dynamic systems over the real erasure channel. In [17], a necessary and sufficient condition for real - time reliable data reconstruction of noiseless nonlinear dynamic systems over Additive White Gaussian Noise (AWGN) channel is presented. [18] presents a new technique for mean square asymptotic reference tracking of nonlinear dynamic systems over AWGN channel. The nonlinear dynamic system considered in [18] has periodic outputs to sinusoidal inputs and is cascaded with a bandpass filter acting as encoder. The authors in [19] also studied the stability problem of nonlinear constrained systems over the real erasure channel subject to random dropouts and delays. In [20], the authors designed state tracker for nonlinear networked control systems over the FlexRay. [21] presents a second order sliding mode control algorithm for a class of nonlinear systems subject to matched uncertainties. The design objective is to reduce data transmission as much as possible over a network subject to loss, jitter and delays, while guaranteeing satisfactory performance in terms of stability and robustness.

1.2 Paper Contributions

The above literature review reveals that the previous works on measurement and control of nonlinear systems over communication channels subject to imperfections are limited to state tracking and/or stability for the digital noiseless channel ([9], [10]), AWGN channel ([17] and [18]), the real erasure channel (e.g., [15], [19]) or the

nonlinear Lipschitz dynamic systems (e.g., [13]). Nevertheless, recently the problem of state tracking, stability and in particular reference tracking of autonomous vehicles (miniature drones, autonomous road vehicles and autonomous under water vehicles) becomes important. Dynamic of these systems can be represented by the unicycle model [29], which is more complicated to be described by the Lipschitz systems. Also, it is customary to use WiFi for the tactile control of these vehicles, in which this type of communication is modeled by the packet erasure channel with feedback acknowledgment for the miniature sized vehicles. Hence, this paper addresses the problem of state tracking, stability and in particular reference tracking of the nonlinear systems over the packet erasure channel with feedback acknowledgment, as is shown in Fig. 2.
To the best of our knowledge, the problem of reference tracking of nonlinear dynamic systems and in particular the unicycle model over the packet erasure channel, which arises, for example, in the tactile control of miniature drones, has not been addressed before; and for the first time is addressed in this paper. To address this problem, we present a new technique that extends the classical linearization method [32] to the context of networked control systems; which is another main contribution of this paper. In the proposed technique, linearization is applied when the error between system states and an estimate of these states at the decoder increases; and within each linearized zone, the available linear networked control techniques are used. The stability of linear switching system which is resulted from linearizing the nonlinear system is shown. Subsequently, the satisfactory performance of the proposed technique is illustrated by computer simulations by applying this technique on the unicycle model that represents the nonlinear dynamic of the miniature drones, autonomous road vehicles and autonomous under water vehicles.

1.3 Paper Organization

The paper is organized as follows. In Section 2, the problem formulation is presented. Section 3 is devoted to the design of a proper encoder and decoder for tracking state trajectory. Then, in Section 4, the proper controller for reference tracking (and hence the stability) is presented. Section 5 is devoted to simulation results. Finally, the paper is concluded by summarizing the contributions of the paper in Section 6.

2 Problem Formulation

Throughout, certain conventions are used: $E[\cdot]$ denotes the expected value, $|\cdot|$ the absolute value, $\|\cdot\|$ the Euclidean norm and V' denotes the transpose of vector/matrix V. A^{-1} and $\lambda_i(A)$ denote the inverse and eigenvalues of a square matrix A, respectively. '$\doteq$' means 'by definition is equivalent to' and $Z^t \doteq (Z_1, Z_2, ..., Z_t)$.

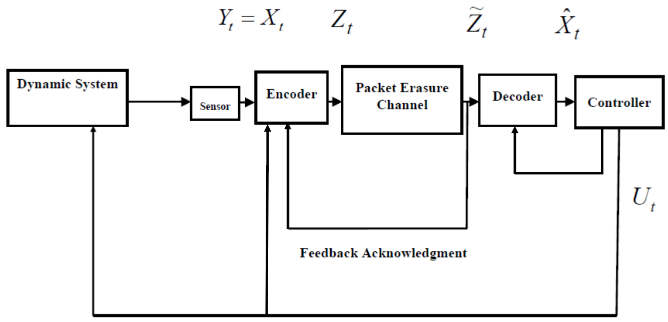

Fig. 2. A dynamic system controlled over the packet erasure channel with feedback acknowledgment.

$\mathbb{R}$ and $\mathbb{N}$ denote the sets of real numbers and natural numbers, respectively; and I is the identity matrix. Also, $X^{(i)}$ denotes the ith element of the vector X and $\underline{0}$ denotes the zero vector/matrix. $N_+ \doteq \{0, 1, 2, 3, ...\}$ and $\mathbb{R}^+$ is the set of non-negative real numbers.
This paper is concerned with almost sure asymptotic tracking of the state trajectory, stability and reference tracking of nonlinear dynamic systems over the packet erasure channel, as is shown in the block diagram of Fig. 2. The building blocks of Fig. 2 are described below:
Dynamic System: The dynamic system is described by the following nonlinear discrete time system:

$$\begin{cases} X_{t+1} = F(X_t, U_t) \\ Y_t = X_t \end{cases} \tag{1}$$

where $t \in N_+$ is the time instant, $F(X_t, U_t) \in \mathbb{R}^n$ is a vector nonlinear function, $X_t \in \mathbb{R}^n$ is the vector of the states of the system, $Y_t \in \mathbb{R}^n$ is the observation signal and $U_t \in \mathbb{R}^m$ is the control signal. Throughout, it is assumed that the probability measure associated with the initial state X_0 with components $X_0^{(i)}$, $i = \{1, 2, ..., n\}$, has bounded support. That is, for each $i \in \{1, 2, ..., n\}$ there exists a compact set $[-L_0^{(i)}, L_0^{(i)}] \in \mathbb{R}$ such that $\Pr(X_0^{(i)} \in [-L_0^{(i)}, L_0^{(i)}]) = 1$. Note that X_0 is unknown for the remote decoder and controller.
Communication Channel: Communication channel between system and controller is a limited capacity erasure channel with feedback acknowledgment. It is a digital channel that transmits a packet of binary data in each channel use. The channel input and output alphabets are denoted by Z and $\tilde{Z}$, respectively; and Z_t denotes the channel input at time instant $t \in N_+$, which is a packet of binary data with length R_t containing information bits. Also $\tilde{Z}_t$ denotes the corresponding channel output. Let e denote the erasure symbol. Then,

$$\tilde{Z}_t = \begin{cases} Z_t & \textit{with probability } 1-\alpha \\ e & \textit{with probability } \alpha \end{cases} \tag{2}$$

That is, this channel erases a transmitted packet with probability α. Throughout, it is assumed that the erasure probability α is known a priori. In the channel considered in this paper, there are feedback acknowledgments from receiver to transmitter. That is, if a transmission is successful, an acknowledgment bit is sent from receiver to transmitter indicating that the transmission was successful. The packet erasure channel with feedback acknowledgment is an abstract model for the commonly used data transmission technologies, such as the Internet and WiFi.

Encoder: Encoder is a causal operator denoted by $Z_t = \mathcal{E}(Y_t, \tilde{Z}^{t-1}, U^{t-1})$ that maps the system output Y_t (by the knowledge of the past channel outputs and control signals) to the channel input Z_t, which is a string of binaries with length R_t. In the closed loop feedback system of Fig. 2, encoder and decoder are used to compensate the effects of the random packet dropout and distortion due to limitation on channel capacity.

Decoder: Decoder is a causal operator denoted by $\hat{X}_t = D(\tilde{Z}^t, U^{t-1})$ that maps the channel output to $\hat{X}_t$ (the estimate of the state variable at the decoder).

Controller: Controller has the following structure $U_t = K_t\hat{\hat{X}}_t + W_t$, where K_t is the controller gain, $\hat{\hat{X}}_t = \hat{X}_t - \mathcal{R}_t$ (where $\mathcal{R}_t$ is the reference signal) and W_t is used for reference tracking and will be determined shortly.

The objective of this paper is to design an encoder, decoder and a controller that result in almost sure asymptotic tracking of the state trajectory, stability and reference tracking of the system (1), as defined below:

Definition 2.1 *(Almost Sure Asymptotic Tracking of the State Trajectory): Consider the block diagram of Fig. 2 described by the nonlinear dynamic system (1) over the packet erasure channel, as described above. It is said that the state trajectory is almost sure asymptotically tracked if there exist an encoder and a decoder such that the following property holds:* $\Pr(\lim_{t\to\infty} \|X_t - \hat{X}_t\| = 0) = 1$.

Definition 2.2 *(Almost Sure Asymptotic Stability): Consider the block diagram of Fig. 2 described by the nonlinear dynamic system (1) over the packet erasure channel, as described above. It is said that the system is almost sure asymptotically stable if there exist an encoder, decoder and a controller such that the following property holds:* $\Pr(\lim_{t\to\infty} \|X_t\| = 0) = 1$.

Definition 2.3 *(Almost Sure Asymptotic Reference Tracking): Consider the block diagram of Fig. 2 described by the nonlinear dynamic system (1) over the packet erasure channel, as described above. It is said that the system is almost sure asymptotically track the reference signal* $\mathcal{R}_t \in \mathbb{R}^n$ *if there exist an encoder, decoder and a controller such that the following property holds:* $\Pr(\lim_{t\to\infty} \|X_t - \mathcal{R}_t\| = 0) = 1$.

Remark 2.4 *Note that the stability is a special case of the reference tracking with* $\mathcal{R}_t = \underline{0}$.

3 Encoder and Decoder for Tracking the State Trajectory

In this section, for the simplicity of presentation, we first address the state tracking problem for the linear systems; and then, we extend the results to the nonlinear case. We start from the linear scalar case.

3.0.1 Linear Scalar Case

Suppose that the dynamic system is linear and scalar ($X_t \in \mathbb{R}$). We present an encoder, decoder and a necessary and sufficient condition on the length of transmitted packets R at each time instant, under which the dynamic system (3) in the block diagram of Fig. 2 almost sure asymptotically track the state trajectory.

$$\begin{cases} X_{t+1} = AX_t + BU_t, \ X_0 \in [-L_0, L_0] \subset \mathbb{R} \\ \quad Y_t = X_t \end{cases} \tag{3}$$

Encoding Scheme: At time instant $t = 0$, we notice that $X_0 \in [-L_0, L_0]$. At this time instant, the encoder and decoder partition the interval $[-L_0, L_0]$ into 2^R equal sized, non-overlapping sub-intervals and the center of each sub-interval is chosen as the index of that interval which are denoted by $\gamma_0, \gamma_1, ..., \gamma_{2^R-1}$. When the encoder observes the initial state X_0, the index of the sub-interval that includes X_0 is encoded into R bits and transmitted to the decoder through the packet erasure channel. If the decoder receives this R bits successfully, it identifies the index of the sub-interval where X_0 lives in (e.g., γ_{j0} where $j0 \in \{0, 1, ..., 2^R - 1\}$); and the value of this index is chosen as $\hat{X}_0$ which is the estimate of X_0 at the decoder (i.e., $\hat{X}_0 = \gamma_{j0}$). Therefore, the estimation error for this case is bounded above by $|X_0 - \hat{X}_0| \leq V_0 = \frac{L_0}{2^R}$. But if erasure occurs, then $\hat{X}_0 = 0$; and therefore, $|X_0 - \hat{X}_0| \leq V_0 = L_0$. Hence, the estimation error is bounded above by

$$|X_0 - \hat{X}_0| \leq V_0 = M_0 L_0;$$
$$M_0 = \begin{cases} \frac{1}{2^R}, & \Pr(M_0 = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_0 = 1) = \alpha \end{cases} \tag{4}$$

Note that M_0 is the indicator of successful transmission or failed transmission at the time instant $t = 0$. Also, note that as the encoder has access to the feedback acknowledgment, it knows the value of M_0 for the time instant $t = 1$.

At the time instant $t = 1$, the encoder encodes $X_1 - A\hat{X}_0 - BU_0$. To encode this signal, the interval $[-L_1, L_1]$ is calculated as follows: $|X_1 - A\hat{X}_0 - BU_0| = |AX_0 - A\hat{X}_0| = |A||X_0 - \hat{X}_0| \leq |A|V_0 = L_1$. Then, similar to the previous time instant, the encoder and decoder partition the interval $[-L_1, L_1]$ into 2^R equal sized, non-overlapping sub-intervals; and the center of each sub-interval is chosen as the index of that interval. When the encoder observes the signal $X_1 - A\hat{X}_0 - BU_0$, the index of the sub-interval that includes $X_1 - A\hat{X}_0 - BU_0$ (e.g., γ_{j1}) is encoded into R bits and transmitted to the decoder through the packet erasure channel. Then, the decoder constructs $\hat{X}_1$ as below:

$$\hat{X}_1 = \begin{cases} \gamma_{j1} + A\hat{X}_0 + BU_0, & \Pr(M_1 = \frac{1}{2^R}) = 1 - \alpha \\ A\hat{X}_0 + BU_0, & \Pr(M_1 = 1) = \alpha \end{cases} \tag{5}$$

and therefore,

$$|X_1 - \hat{X}_1| \leq V_1 = M_1 L_1;$$
$$M_1 = \begin{cases} \frac{1}{2^R}, & \Pr(M_1 = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_1 = 1) = \alpha \end{cases} \tag{6}$$

Similarly, for time instants $t > 1$, the encoder encodes $X_t - A\hat{X}_{t-1} - BU_{t-1}$. To encode this signal the interval $[-L_t, L_t]$ is calculated as follows: $|X_t - A\hat{X}_{t-1} - BU_{t-1}| = |AX_{t-1} - A\hat{X}_{t-1}| = |A||X_{t-1} - \hat{X}_{t-1}| \leq |A|V_{t-1} \doteq L_t$. Then, the encoder and decoder partition the interval $[-L_t, L_t]$ into 2^R equal sized, non-overlapping sub-intervals and the center of each sub-interval is chosen as the index of that interval. When the encoder observes the signal $X_t - A\hat{X}_{t-1} - BU_{t-1}$, the index of the sub-interval that includes $X_t - A\hat{X}_{t-1} - BU_{t-1}$ (e.g., γ_{jt}) is encoded into R bits and transmitted to the decoder through the packet erasure channel. Then, the decoder constructs $\hat{X}_t$ as follows: $\hat{X}_t = \gamma_{jt} + A\hat{X}_{t-1} + BU_{t-1}$, if $M_{t-1} = \frac{1}{2^R}$ with the probability of $\Pr(M_{t-1} = \frac{1}{2^R}) = 1 - \alpha$; and $\hat{X}_t = A\hat{X}_{t-1} + BU_{t-1}$, if $M_{t-1} = 1$ with the probability of $\Pr(M_{t-1} = 1) = \alpha$. Therefore,

$$|X_t - \hat{X}_t| \leq V_t = M_t L_t;$$
$$M_t \doteq \begin{cases} \frac{1}{2^R}, & \Pr(M_t = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_t = 1) = \alpha \end{cases} \tag{7}$$

By following a similar procedure, as described above, the sequence $\hat{X}_0, \hat{X}_1, \hat{X}_2, ...$ are constructed at the decoder. As shown in [4], the above coding scheme results in almost sure asymptotic tracking of the state trajectory. This result is presented in the following proposition.

Proposition 3.1 *Consider the control system of Fig. 2 described by the dynamic system (3) over the packet erasure channel with erasure probability α and feedback acknowledgment, as described earlier. Suppose that the packet length R satisfies the following inequality:*

$$(1 - \alpha)R > \max\{0, \log_2 |A|\} \tag{8}$$

Then, using the above encoding and decoding scheme, we have almost sure asymptotic tracking of the state trajectory in the form of $\hat{X}_t \to X_t$, P-a.s.; or equivalently, $\Pr(\lim_{t\to\infty} \|X_t - \hat{X}_t\| = 0) = 1$.

Proof: See [4].

Remark 3.2 *As it is shown in [4], the condition $(1 - \alpha)R \geq \max\{0, \log_2 |A|\}$ is also a necessary condition for almost sure asymptotic tracking of the state trajectory. Therefore, $\frac{1}{1-\alpha}\max\{0, \log_2 |A|\}$ is the minimum bit length for almost sure asymptotic tracking of the state trajectory of the system (3) over the packet erasure channel.*

3.0.2 Linear Vector Case

Now, suppose the dynamic system is linear and $X_t \in \mathbb{R}^n$. Due to the space limitation, in the following we just explain how we can extend the above coding scheme for this case. By implementing a proper similarity transformation to the system matrix A, we can always transform this matrix to the real Jordan form [5]. As a result of this transformation, the system (3) is decomposed into several decoupled subsystems, in which for each subsystem, the previous encoder and decoder can be implemented separately. Consequently, under the condition $(1-\alpha)R > \sum_{i=1}^{n} \max\{0, \log_2 |\lambda_i(A)|\}$, it is easily shown that almost sure tracking of the state trajectory is achieved for this case.

3.0.3 Nonlinear Case

In this section, for the simplicity of presentation we first consider the dynamic system (1) with $X_t \in \mathbb{R}$. Using the linearization method [32], we extend the above results to nonlinear systems. We present an encoder, decoder and a sufficient condition on the length of transmitted packets R_t at each time instant that guarantee almost sure asymptotic state tracking of the family of the equivalent linear dynamic models which is resulted from linearizing the nonlinear dynamic system (1). As in each linearized zone, we deal with a new linear dynamic system, the length of transmitted packet may be different in different zones. Hence, we denote the length of transmitted packet in the linearized zone j by $R_{[j]}$. At the time instant $t = 0$, we notice that $X_0 \in [-L_0, L_0]$; and we fix the rate to be $\bar{R}_{[0]}$. Then, using the above method for the linear systems, $\hat{\bar{X}}_0$ is reconstructed. Then, at this time instant, the encoder and decoder linearize the nonlinear dynamic system at point $\hat{\bar{X}}_0$, which results in a state space system matrix $A_{[0]}$ and $B_{[0]}$ of the equivalent linear model. Then, similar to the linear case, the encoder and decoder partition the interval $[-L_0, L_0]$ into $2^{R_{[0]}}$ equal sized, non-overlapping sub-intervals and the center of each sub-interval is chosen as the index of that interval $(\gamma_0, \gamma_1, ..., \gamma_{2^{R_{[0]}}-1})$. Subsequently, the index of the sub-interval that includes X_0 (e.g., γ_{j0} where

$j0 \in \{0, 1, ..., 2^{R_{[0]}} - 1\}$) is encoded into $R_{[0]}$ bits and transmitted to the decoder through the packet erasure channel. If the decoder receives this $R_{[0]}$ bits successfully, it identifies the index of the sub-interval where X_0 lives in; and the value of this index is chosen as $\hat{X}_0$. Therefore, the decoding error for this case is bounded above by $|X_0 - \hat{X}_0| \leq V_0 = \frac{L_0}{2^{R_{[0]}}}$. But if erasure occurs, then $\hat{X}_0 = 0$; and therefore, $|X_0 - \hat{X}_0| \leq V_0 = L_0$. Hence, the decoding error can be represented as follows:

$$|E_0| \doteq |X_0 - \hat{X}_0| \leq V_0 = M_0 L_0;$$
$$M_0 = \begin{cases} \frac{1}{2^{R_{[0]}}}, & \Pr(M_0 = \frac{1}{2^{R_{[0]}}}) = 1 - \alpha \\ 1, & \Pr(M_0 = 1) = \alpha \end{cases} \tag{9}$$

At the time instant $t = 1$, the encoder encodes $X_1 - A_{[0]}\hat{X}_0 - B_{[0]}U_0$. To encode this signal, the interval $[-L_1, L_1]$ is calculated as follows:$|X_1 - A_{[0]}\hat{X}_0 - B_{[0]}U_0| = |A_{[0]}X_0 - A_{[0]}\hat{X}_0| = |A_{[0]}||X_0 - \hat{X}_0| \leq |A_{[0]}|V_0 = L_1$. Then, encoder and decoder partition the interval $[-L_1, L_1]$ into $2^{R_{[0]}}$ equal sized, non-overlapping sub-intervals and the center of each sub-interval is chosen as the index of that interval. When the encoder observes the signal $X_1 - A_{[0]}\hat{X}_0 - B_{[0]}U_0$, the index of the sub-interval that includes $X_1 - A_{[0]}\hat{X}_0 - B_{[0]}U_0$ (e.g., γ_{j1} where $j1 \in \{0, 1, ..., 2^{R_{[0]}} - 1\}$) is encoded into $R_{[0]}$ bits and transmitted to the decoder through the packet erasure channel. Subsequently, the decoder constructs $\hat{X}_1$ as follows: $\hat{X}_1 = \gamma_{j1} + A_{[0]}\hat{X}_0 + B_{[0]}U_0$, if $M_1 = \frac{1}{2^{R_{[0]}}}$ with the probability of $\Pr(M_1 = \frac{1}{2^{R_{[0]}}}) = 1 - \alpha$; and $\hat{X}_1 = A_{[0]}\hat{X}_0 + B_{[0]}U_0$, if $M_1 = 1$ with the probability of $\Pr(M_1 = 1) = \alpha$. Therefore,

$$|E_1| \doteq |X_1 - \hat{X}_1| \leq V_1 = M_1 L_1;$$
$$M_1 = \begin{cases} \frac{1}{2^{R_{[0]}}}, & \Pr(M_1 = \frac{1}{2^{R_{[0]}}}) = 1 - \alpha \\ 1, & \Pr(M_1 = 1) = \alpha \end{cases} \tag{10}$$

For the next step ($t = 2$), if $|E_1| \leq |E_0|$, this procedure, as described above, continues with the equivalent state space matrices $A_{[0]}$ and $B_{[0]}$ and the packet length $R_{[0]}$; but if $|E_1| > |E_0|$, then the encoder linearizes the nonlinear dynamic system at the new point $\hat{X}_1$ that results in the state space system matrices $A_{[1]}$ and $B_{[1]}$ of the equivalent linear model (i.e., the system (11) with $j = 1$). The encoder by sending $R_{[1]} \neq R_{[0]}$ bits through the packet erasure channel announces that a new linearization is applied. Therefore, the decoder performs the same linearization; and the rest of procedure is continued with new matrices $A_{[1]}$ and $B_{[1]}$.

$$\begin{cases} X_{t+1} = A_{[0]}X_t + B_{[0]}U_t; \ t \in [0, t_1), \\ X_{t+1} = A_{[1]}X_t + B_{[1]}U_t; \ t \in [t_1, t_2) \\ \vdots \\ X_{t+1} = A_{[j]}X_t + B_{[j]}U_t; \ t \in [t_j, t_{j+1}), \quad j \in N_+ \\ Y_t = X_t \end{cases} \tag{11}$$

By following a similar procedure, as described above, the sequence $\hat{X}_0, \hat{X}_1, \hat{X}_0, ...$ are constructed at the decoder. The vector case ($X_t \in \mathbb{R}^n$) can be treated similarly. Now, we must show that the above coding scheme results in almost sure asymptotic tracking of the state trajectory. This result is shown in the following proposition.

Proposition 3.3 *Consider the control system of Fig. 2 described by the dynamic system (1) over the packet erasure channel with erasure probability $\alpha \neq 1$, as described earlier. Suppose that $\triangle t_j \doteq t_{j+1} - t_j$ ($j \in N_+$)s are sufficiently large (as will be explained later) and the packet lengths $R_t = R_{[j]}$ ($t \in [t_j, t_{j+1})$) satisfy the following inequalities:*

$$(1 - \alpha)R_{[j]} > \sum_{i=1}^{n} \max\{0, \log_2 |\lambda_i(A_{[j]})|\}; \forall j \in N_+ \tag{12}$$

Then, using the proposed encoding and decoding scheme, we have almost sure asymptotic tracking of the state trajectory in the form of $\Pr(\lim_{t\to\infty} \|X_t - \hat{X}_t\| = 0) = 1$.

Proof: : For the simplicity of presentation, first consider the scalar case. Choose any packet lengths $R_{[j]}$ that satisfy the condition (12). For these lengths, define the random variable M_t ($t \in [t_j, t_{j+1})$) as follows:

$$M_t = \begin{cases} \frac{1}{2^{R_{[j]}}}, & \Pr(M_t = \frac{1}{2^{R_{[j]}}}) = 1 - \alpha \\ 1, & \Pr(M_t = 1) = \alpha \end{cases} \tag{13}$$

This random variable is the indicator of successful transmission or failed transmission at time instant t. Using the above encoding and decoding scheme, we have

$$|X_0 - \hat{X}_0| \leq V_0 = M_0 L_0$$

$$|X_1 - \hat{X}_1| \leq V_1 = M_1 L_1 = M_1 |A_{[0]}| V_0$$

$$\vdots$$

$$|X_{t_1-1} - \hat{X}_{t_1-1}| \leq V_{t_1-1} = M_{t_1-1} L_{t_1-1} =$$
$$M_{t_1-1}|A_{[0]}|V_{t_1-2} = ... = M_{t_1-1}|A_{[0]}...M_1|A_{[0]}|V_0$$

$$\Rightarrow V_{t_1-1} = \frac{L_0}{|A_{[0]}|}\prod_{t_0=0}^{t_1-1} M_k|A_{[0]}| =$$

$$\frac{L_0}{A_{[0]}}(2^{(t_1-1)}\frac{1}{t_1-1}\sum_{k=t_0}^{t_1-1}\log_2(M_k|A_{[0]}|)) \qquad (14)$$

$$|X_{t_1} - \hat{X}_{t_1}| \leq V_{t_1} = M_{t_1}L_{t_1},\ (L_{t_1} = |A_{[1]}|V_{t_1-1})$$

$$\vdots$$

$$|X_{t_2-1} - \hat{X}_{t_2-1}| \leq V_{t_2-1} = M_{t_2-1}L_{t_2-1} = M_{t_2-1}|A_{[1]}|$$
$$.V_{t_2-2} = ... = M_{t_2-1}|A_{[1]}|M_{t_2-2}|A_{[1]}|...M_{t_1}|A_{[1]}|V_{t_1-1}$$

$$\Rightarrow V_{t_2-1} = V_{t_1-1}\prod_{k=t_1}^{t_2-1} M_k|A_{[1]}| =$$

$$V_{t_1-1}(2^{(t_2-t_1)}\frac{1}{t_2-t_1}\sum_{k=t_1}^{t_2-t_1}\log_2(M_k|A_{[1]}|)) \qquad (15)$$

$$\vdots$$

$$|X_{t_j-1} - \hat{X}_{t_j-1}| \leq V_{t_j-1} = ... = M_{t_j-1}|A_{[j-1]}|M_{t_j-2}$$
$$.|A_{[j-1]}|...M_{t_{j-1}}|A_{[j-1]}|V_{t_{j-1}-1}$$

$$\Rightarrow V_{t_j-1} = V_{t_{j-1}-1}\prod_{k=t_{j-1}}^{t_j-1} M_k|A_{[j-1]}| =$$

$$V_{t_{j-1}-1}(2^{(t_j-t_{j-1})}\frac{1}{t_j-t_{j-1}}\sum_{k=t_{j-1}}^{t_j-t_{j-1}}\log_2(M_k|A_{[j-1]}|)) \qquad (16)$$

$$\vdots$$

where $2 < ... < j(\in N_+)$ and $0 \ll t_1 \ll t_2 \ll ... \ll t_j$ are time instants that the decoding error is increasing and new linearization at these points is applied.
Now, as $\triangle t_j = t_{j+1} - t_j$ $(j \in N_+)$s are sufficiently large, the strong law of large numbers [33] is valid (conditions for having this assumption are given in Remark 3.4).
The strong law of large numbers states that if the random process M_t is an i.i.d. (independent identically distributed), then $\frac{1}{n}\sum_{t=0}^{n-1} M_t \to E[M_0]$, P-a.s. Therefore, for V_{t_1-1} by the strong law of large numbers, we have $(t_0 = 0)$:

$$\frac{1}{\Delta t_0}\sum_{k=t_0}^{t_1-1}\log_2(M_k|A_{[0]}|) \to E[\log_2(M_{t_0}|A_{[0]}|)]$$
$$= (1-\alpha)\log_2(\frac{1}{2^{R_{[0]}}}|A_{[0]}|) + \alpha\log_2(|A_{[0]}|) \qquad (17)$$

Now, as the transmitted packet length $R_{[0]}$ was chosen such that

$$(1-\alpha)R_{[0]} > \max\{0, \log_2|A_{[0]}|\}, \qquad (18)$$

the following inequality holds

$$(1-\alpha)\log_2(\frac{1}{2^{R_{[0]}}}|A_{[0]}|) + \alpha\log_2(|A_{[0]}|) < 0. \qquad (19)$$

Consequently, as Δt_0 is sufficiently large, from (19), (18) and (14) it follows that $V_{t_1-1} < \frac{L_0}{A_{[0]}}$ (and $V_{t_1-1} \to 0$ as $\Delta t_0 \to \infty$).
Similarly, as $\Delta t_1 = t_2 - t_1$ is sufficiently large, from the strong law of large numbers for the expression $\frac{1}{t_2-t_1}\sum_{k=t_1}^{t_2-1}\log_2(M_k|A_{[1]}|)$, we have

$$\frac{1}{\Delta t_1}\sum_{k=t_1}^{t_2-1}\log_2(M_k|A_{[1]}|) \to E[\log_2(M_{t_1}|A_{[1]}|)]$$
$$= (1-\alpha)\log_2(\frac{1}{2^{R_{[1]}}}|A_{[1]}|) + \alpha\log_2(|A_{[1]}|) \qquad (20)$$

Now, as the length $R_{[1]}$ was chosen such that

$$(1-\alpha)R_{[1]} > \max\{0, \log_2|A_{[1]}|\} \qquad (21)$$

the following inequality holds

$$(1-\alpha)\log_2(\frac{1}{2^{R_{[1]}}}|A_{[1]}|) + \alpha\log_2(|A_{[1]}|) < 0. \qquad (22)$$

Consequently, as Δt_1 is sufficiently large, from (22), (21) and (15) it follows that $V_{t_2-1} < V_{t_1-1}$ (and $V_{t_2-1} \to 0$ as $\Delta t_1 \to \infty$).
Recall that the strong law of large numbers [33] states that if the random process M_t is i.i.d, then $\frac{1}{n}\sum_{t=0}^{n-1} M_t \to E[M_0]$, P-a.s. Having that as M_t given in (13) is an i.i.d. process, we have

$$\lim_{\Delta t_{j-1}\to\infty}(\frac{1}{\Delta t_{j-1}}\sum_{k=t_{j-1}}^{t_j-1}\log_2(M_k|A_{[j-1]}|)) =$$
$$E[\log_2(M_{t_{j-1}}|A_{[j-1]}|)] = (1-\alpha)\log_2(\frac{1}{2^{R_{[j-1]}}}|A_{[j-1]}|)$$
$$+ \alpha\log_2(|A_{[j-1]}|) \qquad (23)$$

Now, as the packet length $R_{[j-1]}$ was chosen such that

$$(1-\alpha)R_{[j-1]} > \max\{0, \log_2|A_{[j-1]}|\} \geq \log_2|A_{[j-1]}| \qquad (24)$$
$$\Rightarrow -(1-\alpha)R_{[j-1]} + \log_2|A_{[j-1]}| < 0$$
$$\Rightarrow (1-\alpha)\log_2\frac{1}{2^{R_{[j-1]}}} + (1-\alpha+\alpha)\log_2|A_{[j-1]}| < 0$$

we have:

$$(1-\alpha)\log_2(\frac{1}{2^{R_{[j-1]}}}|A_{[j-1]}|) + \alpha\log_2(|A_{[j-1]}|) < 0. \qquad (25)$$

Consequently, from (25), (23) and (16) it follows that $V_{t_j-1} < V_{t_{j-1}-1}$ (and $V_{t_j-1} \to 0$ as $\Delta t_{j-1} \to \infty$).

Now, as we have shown that

$$V_{t_1-1} < \frac{L_0}{A_{[0]}},\ V_{t_2-1} < V_{t_1-1},\ ...$$

we can conclude that

$$0 \leq ... < V_{t_j-1} < ... < V_{t_3-1} < V_{t_2-1} < V_{t_1-1} < \frac{L_0}{A_{[0]}}$$

That is, the sequence $V_{t_j-1} \geq 0$ is a strictly decreasing sequence; and hence, $V_{t_j-1} \to 0$ as $j \to \infty$; and therefore, $|X_{t_j-1} - \hat{X}_{t_j-1}| \to 0$, P-a.s., as $j \to \infty$. Subsequently, we have $0 \leftarrow |X_{t_{j-1}} - \hat{X}_{t_{j-1}}| \leq ... \leq |X_{t_j-1} - \hat{X}_{t_j-1}| \leq |X_{t_j} - \hat{X}_{t_j}| \to 0$, P-a.s. as $j \to \infty$. That is, $\hat{X}_t \to X_t$ as $t \to \infty$, P-a.s. This completes the proof for the scalar case. For the vector case by following the similar procedure it can be shown that state trajectory are asymptotically tracked if the following condition holds: $(1-\alpha)R_{[j]} > \sum_{i=1}^{n} \max\{0, \log_2 |\lambda_i(A_{[j]})|\}, \forall j \in N_+$.

Remark 3.4 *To make $\triangle t_j$ sufficiently large, the following conditions must be satisfied:*
If switching between linear systems is too fast, then the combined system may be unstable [34]. In fact, as shown in [35] to avoid this situation, the average dwell time τ_a, which is a measure of the frequency of switches, must be greater than or equal to a critical value, denoted by τ_a^*. That is,

$$\tau_a \geq \tau_a^*;\ \ \tau_a = \frac{t}{N_t},\ \ \tau_a^* = \frac{\ln h}{\ln\lambda - \ln\lambda^*} \tag{26}$$

where N_t is the number of switches that occurs in the time interval of $[0,t]$ and h, λ and λ^* are defined as follows: For all linearized models, there exist $\lambda_1 < 1$ and $\lambda_2 > 1$ such that the following relations hold:

$$\begin{aligned} \|A_{[j]}\| < 1:\ \ \|A_{[j]}^t\| \leq h_j\lambda_1^t \\ \|A_{[j]}\| \geq 1:\ \ \|A_{[j]}^t\| \leq h_j\lambda_2^t \end{aligned} \tag{27}$$

Then, $h = \max_j h_j$, $\lambda \in [\lambda_1, 1]$ and $\lambda^* \in [\lambda_1, \lambda]$ is the biggest value that satisfies the following inequality for some $c > 0$:

$$\|X_t\| \leq c(\lambda^*)^t \|X_0\|. \tag{28}$$

To satisfy the above condition, the sampling period must be chosen sufficiently small.

4 Reference Tracking of Nonlinear Dynamic Systems

Now, we extend the results to account for reference tracking (and hence stability). To obtain the control signal for tracking the reference signal $\mathcal{R}_t$, we use the proposed coding scheme which results in almost sure asymptotic tracking of the state trajectory. To achieve the reference tracking, we construct a new state space model with the state variable $\tilde{X}_t = X_t - \mathcal{R}_t$, which we make it zero. Throughout, it is assumed that the reference signals $\mathcal{R}_t$ and $\mathcal{R}_{t+1}$ are known a priori at the time instant t, which is not a restrictive assumption.
This result is presented in the following proposition. Before that we need the following lemma.

Lemma 4.1 *Let A be a stable matrix. Let B_t be a set of matrices such that $\|B_t\| \leq L < \infty$ and $\lim_{t\to\infty} \|B_t\| = 0$. Let $S_t = \sum_{k=0}^{t-1} A^{t-1-k}B_k$. Then, $\lim_{t\to\infty} \|S_t\| = 0$ (***Proof:** *See [4]).*

Proposition 4.2 *For the nonlinear case, suppose that for each linearized equivalent system, the matrix $B_{[j]}$ has Pseudo inverse and there exists a matrix $K_{[j]}$ such that the matrix $A_{[j]} + B_{[j]}K_{[j]}$ is a stable matrix (i.e., the pair $(A_{[j]}, B_{[j]})$ is stabilizable). Then, using the proposed coding scheme and the controller $U_t = K_{[j]}\hat{\tilde{X}}_t + W_{[j],t}$ where $W_{[j],t} \doteq -B_{[j]}^{'}(B_{[j]}B_{[j]}^{'})^{-1}(A_{[j]}\mathcal{R}_t - \mathcal{R}_{t+1})$ and $\hat{\tilde{X}}_t \doteq \hat{X}_t - \mathcal{R}_t$ for each linearized system, we have almost sure asymptotic reference tracking, provided Δt_js are sufficiently large and the erasure probability $\alpha \neq 1$.*

Proof: For the nonlinear case, it is shown in the following that for each equivalent linear system, the new state space model is asymptotically stable:

$$\begin{aligned} \tilde{X}_{t+1} &= X_{t+1} - \mathcal{R}_{t+1},\ t \in [t_j, t_{j+1}), && (29)\\ &= A_{[j]}X_t + B_{[j]}U_t - \mathcal{R}_{t+1} + A_{[j]}\mathcal{R}_t - A_{[j]}\mathcal{R}_t && (30)\\ &= A_{[j]}(X_t - \mathcal{R}_t) + B_{[j]}U_t + A_{[j]}\mathcal{R}_t - \mathcal{R}_{t+1} && (31)\\ &= A_{[j]}\tilde{X}_t + B_{[j]}U_t + A_{[j]}\mathcal{R}_t - \mathcal{R}_{t+1} && (32) \end{aligned}$$

By choosing $U_t = K_{[j]}\hat{\tilde{X}}_t + W_{[j],t}$ (where $W_{[j],t} \doteq -B_{[j]}^{'}(B_{[j]}B_{[j]}^{'})^{-1}(A_{[j]}\mathcal{R}_t - \mathcal{R}_{t+1})$) as the control signal we have:

$$\begin{aligned} \tilde{X}_{t+1} &= A_{[j]}\tilde{X}_t + B_{[j]}K_{[j]}\hat{\tilde{X}}_t - B_{[j]}B_{[j]}^{'}(B_{[j]}B_{[j]}^{'})^{-1} \\ &\quad .(A_{[j]}\mathcal{R}_t - \mathcal{R}_{t+1}) + A_{[j]}\mathcal{R}_t \\ &\quad - \mathcal{R}_{t+1} + B_{[j]}K_{[j]}\tilde{X}_t - B_{[j]}K_{[j]}\tilde{X}_t \\ &= (A_{[j]} + B_{[j]}K_{[j]})\tilde{X}_t - B_{[j]}K_{[j]}\tilde{E}_t,\ t \in [t_j, t_{j+1}) \end{aligned} \tag{33}$$

where $\tilde{E}_t \doteq \tilde{X}_t - \hat{\tilde{X}}_t = X_t - \mathcal{R}_t - (\hat{X}_t - \mathcal{R}_t) = X_t - \hat{X}_t = E_t$, $(\hat{\tilde{X}}_t \doteq \hat{X}_t - \mathcal{R}_t)$.
Recall that, as we have shown in Section 3, $E_k (= \tilde{E}_k)$s are bounded and E_k in the jth linearized zone tends to

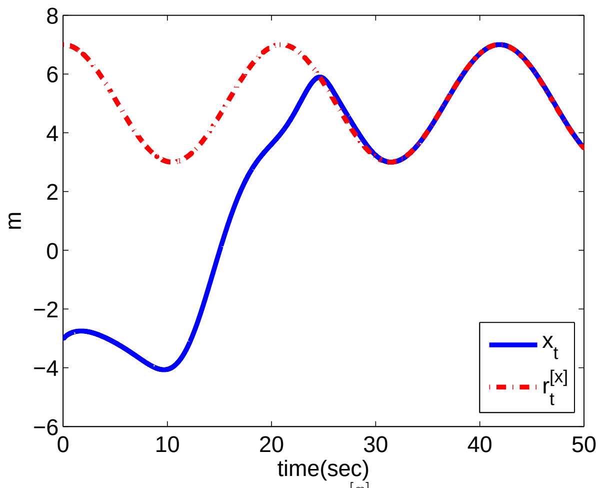

Fig. 3. x_t and $r_t^{[x]}$ for $\alpha = 0.5$.

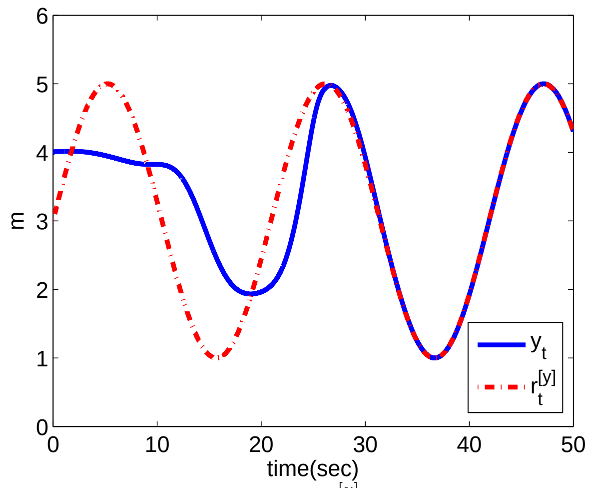

Fig. 4. y_t and $r_t^{[y]}$ for $\alpha = 0.5$.

zero as $\Delta t_j \to \infty$. On the other hand, from (33), we have:

$$\begin{aligned}\tilde{X}_{t_{j+1}-1} &= (A_{[j]} + B_{[j]}K_{[j]})^{t_{j+1}-t_j-1}\tilde{X}_{t_j} \\ &- \sum_{k=t_j}^{t_{j+1}-2} ((A_{[j]} + B_{[j]}K_{[j]})^{t_{j+1}-2-k} B_{[j]}K_{[j]}\tilde{E}_k)\end{aligned} \tag{34}$$

Hence, from Lemma 4.1, we have: $||\tilde{X}_{t_{j+1}-1}|| \to 0$, as $\Delta t_j \to \infty$. This means that at each linearized zone almost sure asymptotic reference tracking is achieved. But, this is not enough for the stability of switching linear system that is resulted from linearizing the nonlinear system. It is shown in [34] if switching between stable linear subsystems is too fast, then the combined system may be unstable. To avoid this situation, as is shown in [35] and discussed in Remark 3.4, the average dwell time τ_a, which is a measure of the frequency of switches, must be greater than or equal to a critical value, denoted by τ_a^*. That is,

$$\tau_a \geq \tau_a^*; \quad \tau_a = \frac{t}{N_t}, \quad \tau_a^* = \frac{\ln h}{\ln \lambda - \ln \lambda^*} \tag{35}$$

where N_t is the number of switches that occurs in the time interval of $[0, t]$ and h, λ and λ^* are defined in Remark 3.4. To satisfy the above condition, the sampling period needs to be chosen small enough. This is equivalent to say that Δt_j is sufficiently large. Having this assumption, the proof is complete.

Remark 4.3 *For each linearized equivalent system, the controller gain $K_{[j]}$ must be chosen so that the matrix $A_{[j]} + B_{[j]}K_{[j]}$ is a stable matrix.*

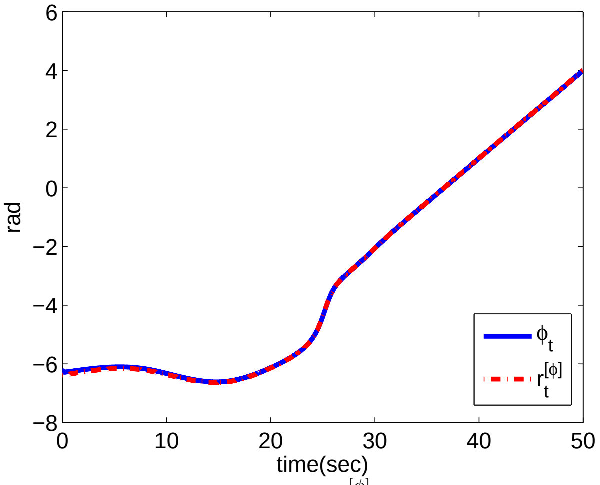

Fig. 5. ϕ_t and $r_t^{[\phi]}$ for $\alpha = 0.5$.

5 Application in Remote Control of Autonomous Vehicles

In this section, for the purpose of illustration, we apply the proposed encoder, decoder and controller to the nonlinear dynamic of miniature drones, autonomous road vehicles or autonomous under water vehicles that can be modeled by the unicycle model [29]. The dynamic of each miniature drones, autonomous road vehicles or autonomous under water vehicles are described by a 6 degrees of freedom model. However, the vehicles dynamic can be handled by local control loops, which results in a kinematic unicycle model, as follows [29]:

$$\begin{cases} \dot{x}(t) = v(t)\cos(\phi(t)) \\ \dot{y}(t) = v(t)\sin(\phi(t)) \\ \dot{\phi}(t) = u(t) \end{cases} \tag{36}$$

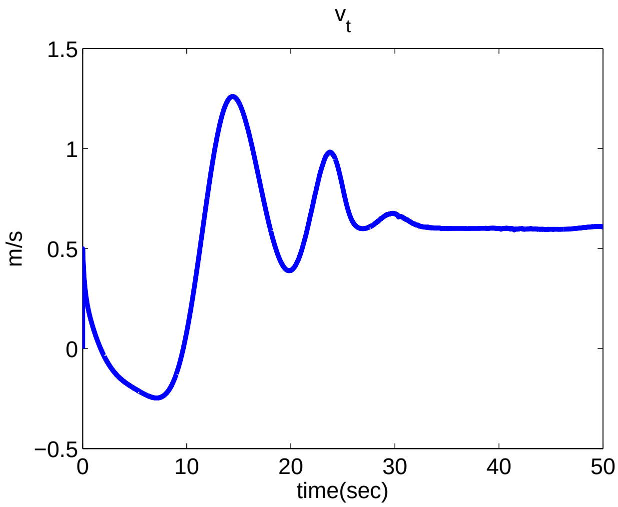

Fig. 6. Control signal: forward velocity.

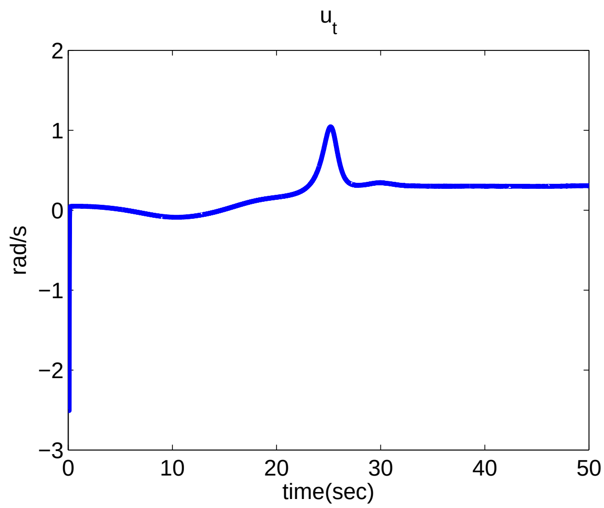

Fig. 7. Control signal: angular velocity.

where $x(t)$, $y(t)$ are the position vector, $\phi(t)$ the heading angle, and the control inputs are the vehicle forward velocity $v(t)$ and the turning rate $u(t)$. The state vector of the system is $X(t) = [x(t), y(t), \phi(t)]'$ and the input vector is $U(t) = [v(t), u(t)]'$. The discrete equivalent model is (37), where T is the sampling period.

$$\begin{cases} x_{t+1} = x_t + Tv_t \cos(\phi_t) \\ y_{t+1} = y_t + Tv_t \sin(\phi_t) \\ \phi_{t+1} = \phi_t + Tu_t \end{cases} \tag{37}$$

where x_t, y_t, ϕ_t, v_t and u_t are the discrete equivalent signals of $x(t)$, $y(t)$, $\phi(t)$, $v(t)$ and $u(t)$, respectively. Note that for this model, the state vector is $X_t = [x_t, y_t, \phi_t]'$. Therefore, the state space representation of the family of the discrete time equivalent linear models is similar

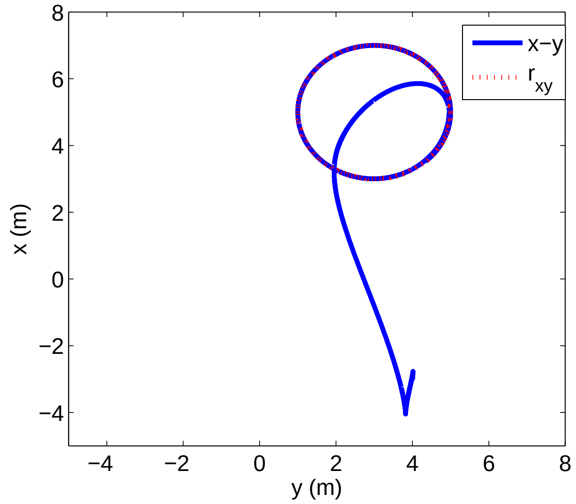

Fig. 8. $x_t - y_t$ diagram for $\alpha = 0.5$.

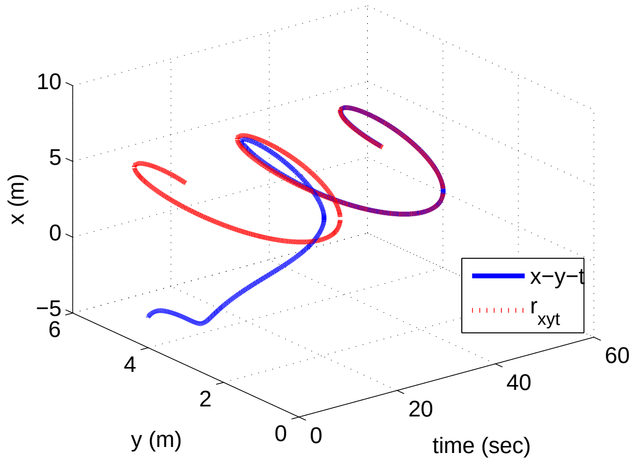

Fig. 9. $x_t - y_t - time$ diagram for $\alpha = 0.5$.

to (11), with the following state space matrices:

$$A_{[j]} = \begin{bmatrix} 1 & 0 & -Tv_{[j]} \sin(\phi_{[j]}) \\ 0 & 1 & Tv_{[j]} \cos(\phi_{[j]}) \\ 0 & 0 & 1 \end{bmatrix} \tag{38}$$

$$B_{[j]} = \begin{bmatrix} T\cos(\phi_{[j]}) & 0 \\ T\sin(\phi_{[j]}) & 0 \\ 0 & T \end{bmatrix} \tag{39}$$

Now, for tracking a circle with the center located at $(5, 3)$ and the radius of 2, by the autonomous vehicle, we choose $\mathcal{R}_t \doteq [r_t^{[x]}, r_t^{[y]}, r_t^{[\phi]}]' = [5 + 2\cos(0.3T\ t), 3 + 2\sin(0.3T\ t), \arctan$ $(\frac{r_t^{[y]} - y_{t-1}}{r_t^{[x]} - x_{t-1}})]'$ as the reference signal. For simulations, we also use $T = 0.01\,\mathrm{sec}$, x_0, $y_0 \in [-10, 10]$,

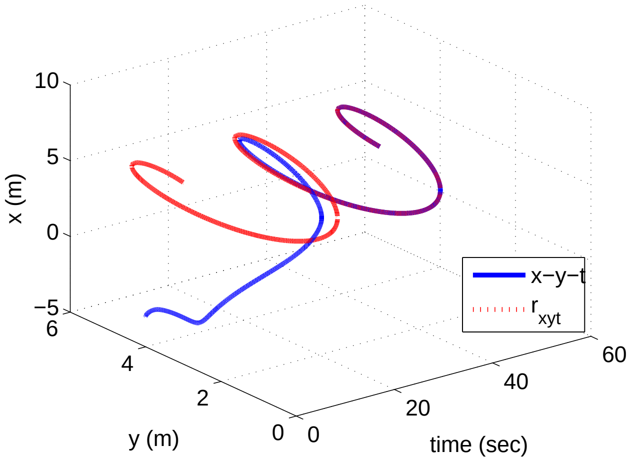

Fig. 10. $x_t - y_t - time$ diagram for $\alpha = 0$.

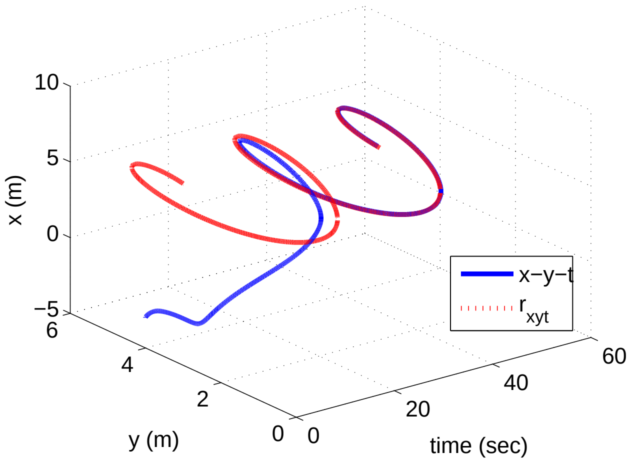

Fig. 11. $x_t - y_t - time$ diagram for $\alpha = 0.9$.

$\phi_0 \in [-2, 2]$ and $\alpha = 0.5$. Fig. 3 to Fig. 9 illustrate the results of the simulations. Note that for a given linearized equivalent system we choose $K_{[j]}$ so that the matrix $A_{[j]} + B_{[j]}K_{[j]}$ is stable (i.e., $K_{[j]} = \begin{bmatrix} -0.025\cos(\phi_{[j]}) & -0.9\sin(\phi_{[j]}) & 0 \\ 0 & 0 & -0.1 \end{bmatrix}$). Fig. 3 to Fig. 9 illustrate that the tracking is achieved. For different probability of the packet dropout α we repeated simulations in Fig. 10 to Fig. 14. Fig. 10 to Fig. 14 illustrate the simulation results for $\alpha = 0$, $\alpha = 0.9$, $\alpha = 0.95$, $\alpha = 0.99$ and $\alpha = 1$, respectively. They illustrate that the performance of the proposed technique is good even for $\alpha = 0.95$. Also, as it is clear from Fig. 14 when the communication channel is completely blocked ($\alpha = 1$), the controller is not able to force the system to track the reference signal. The Root Sum Square Error (RSSE) computed from the sample $t = 30/T$ to the sample $t = 50/T$ (30 sec. to 50 sec.) for different probability of

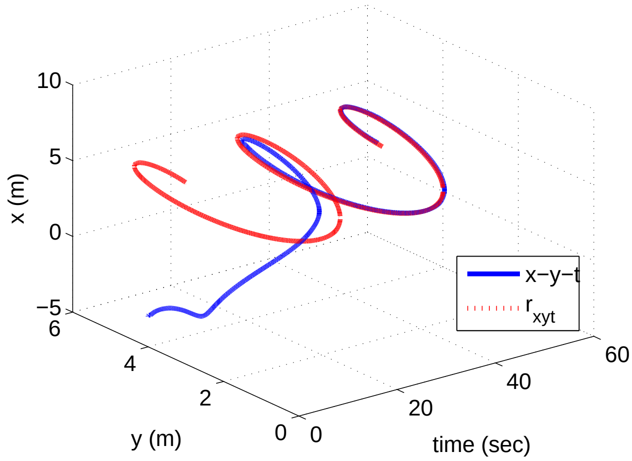

Fig. 12. $x_t - y_t - time$ diagram for $\alpha = 0.95$.

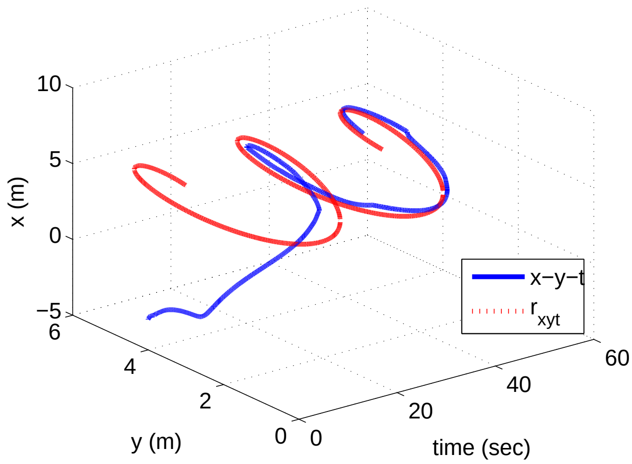

Fig. 13. $x_t - y_t - time$ diagram for $\alpha = 0.99$.

the packet dropout α is shown in the following Table.

α	RSSE
0	1.17
0.5	1.21
0.9	3.42
0.95	8.40
0.99	57.40
1	1144

One of the major assumption of the paper is that Δt_j is sufficiently large; or equivalently, the sampling period T is sufficiently small. To illustrate the importance of this assumption, we repeated simulations in Fig. 15 to Fig. 17 for different Ts and $\alpha = 0.5$. Fig. 9 and Fig. 15

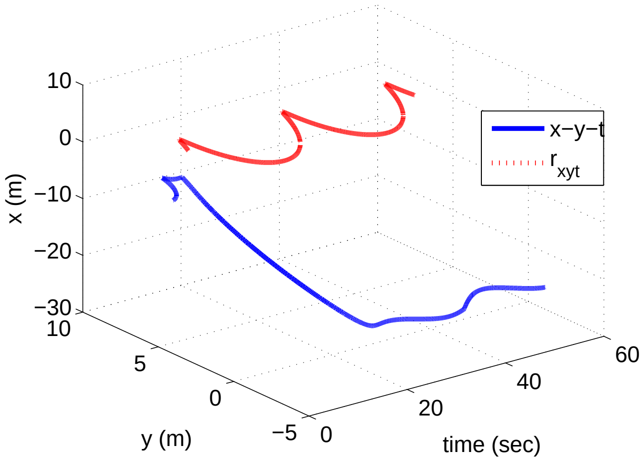

Fig. 14. $x_t - y_t - time$ diagram for $\alpha = 1$.

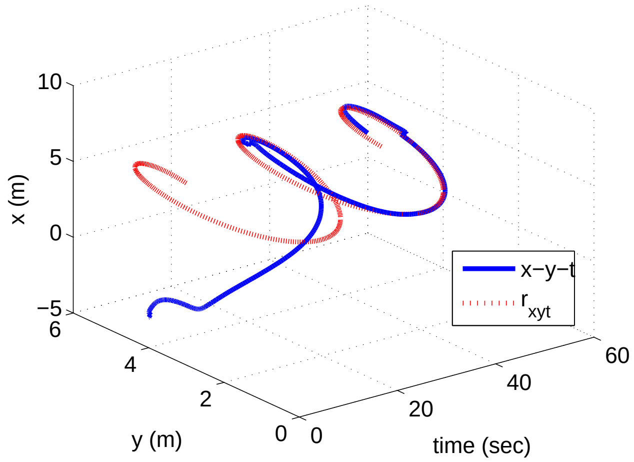

Fig. 15. $x_t - y_t - time$ diagram for $\alpha = 0.5$ and $T = 0.1$.

to Fig. 17 illustrate the simulation results for $T = 0.01$, $T = 0.1$, $T = 0.5$ and $T = 1$, respectively. As is clear from these figures by increasing T, the tracking performance significantly reduces, as expected.

To compare the performance of the proposed technique with the existing techniques, we apply the proposed technique and the feedback linearization control technique of [36] (with the linearized system of (9) and (10) of [36]) to the block diagram of Fig. 2 with the unicycle model of (37) as the dynamical system with the reference signals $r_t^{[x]} = 0.05Tt$ and $r_t^{[y]} = 0.02Tt$ ($T = 0.01\,\mathrm{sec}$) and the following initial conditions: x_0, $y_0 \in [-10, 10]$ and $\phi_0 \in [-2, 2]$. The RSSE computed from the sample $t = 30/T$ to the sample $t = 100/T$ (30 sec . to 100 sec .) for $\alpha = 0.5$, $\alpha = 0.9$ and $\alpha = 0.99$ when the proposed technique is used is shown in the following Table:

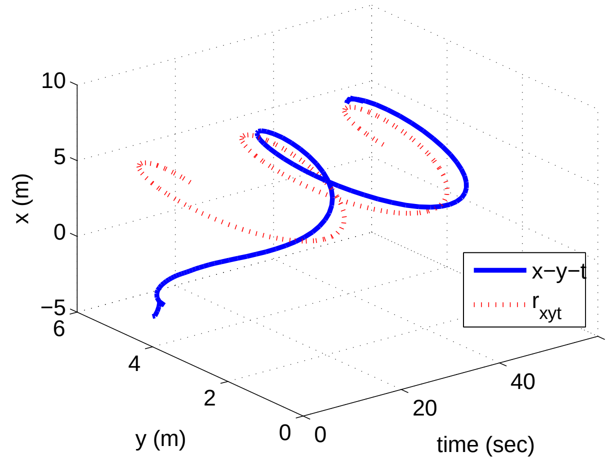

Fig. 16. $x_t - y_t - time$ diagram for $\alpha = 0.5$ and $T = 0.5$.

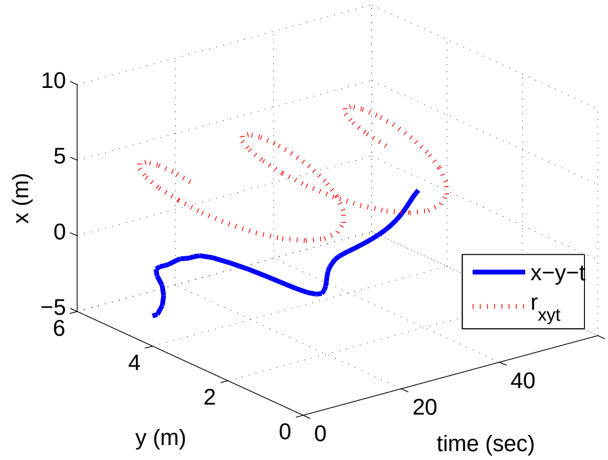

Fig. 17. $x_t - y_t - time$ diagram for $\alpha = 0.5$ and $T = 1$.

α	RSSE
0.5	0.68
0.9	1.03
0.99	13.84

The following table also shows the RSSE computed for the feedback linearization control technique of [36].

α	RSSE
0.5	30.23
0.9	230.75
0.99	636.06

For $\alpha = 0.9$, the performances of the proposed technique and the feedback linearization technique of [36] are illus-

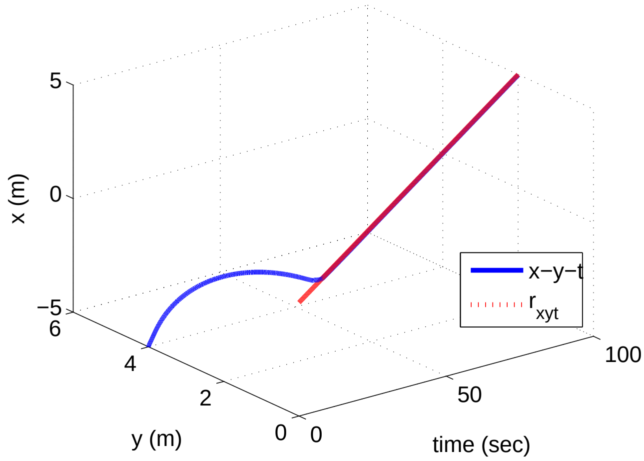

Fig. 18. $x_t - y_t - time$ diagram for $\alpha = 0.9$ and the proposed technique.

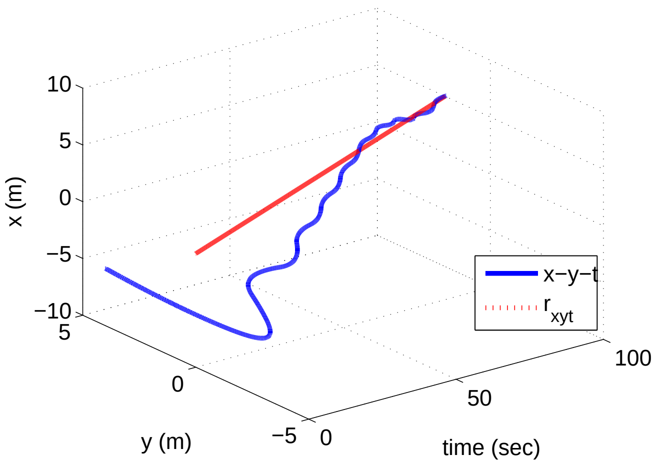

Fig. 19. $x_t - y_t - time$ diagram for $\alpha = 0.9$ and the feedback linearization technique of [36].

trated in Fig. 18 and Fig. 19, respectively. From these tables and figures, it is clear that the proposed technique has a better performance.

6 Conclusion

In this paper a new technique for almost sure asymptotic state and reference tracking of nonlinear dynamic systems, which is based on the linearization method, was presented over the packet erasure channel with feedback acknowledgment. A proper encoder, decoder and controller for tracking the state trajectory and reference tracking of nonlinear systems at the end of communication link was presented when measurements are sent through the packet erasure channel. In the proposed technique, linearization is applied when the state tracking error increases. The satisfactory performance of the proposed reference tracking technique was illustrated by implementing this technique on the unicycle model which represents the nonlinear dynamic of the miniature drones, autonomous road vehicles and autonomous under water vehicles. For future, it is interesting to extend the results to account for stochastic and uncertain nonlinear systems. This research direction is currently underway in our research team.

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