Tele-operation of Autonomous Vehicles over Additive White Gaussian Noise Channel

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Abstract

This paper is concerned with the tele-operation of autonomous vehicles over analog Additive White Gaussian Noise (AWGN) channel, which is subject to transmission noise and power constraint. The nonlinear dynamic of autonomous vehicle is described by the unicycle model and is cascaded with a bandpass filter acting as encoder. Using the describing function method, the nonlinear dynamic of autonomous vehicle is represented by an approximate linear system. Then, the available results for linear control over analog AWGN channel are extended to account for linear continuous time systems with non - real valued and multiple real valued eigenvalues and for tracking a non-zero reference signal. Subsequently, by applying the extended results on the describing function of autonomous vehicles, a mean square control technique including an encoder, decoder and a controller is presented for reference tracking of the tele-operation of autonomous vehicles over AWGN channel. The satisfactory performance of the proposed control technique is illustrated by computer simulations.

Index Terms

Networked control system, tele-operation system, the describing function, the unicycle model, AWGN channel.

I. Introduction

A. Motivation and Background

Tele-operation of autonomous vehicles has become an active research direction in recent years. In these systems, the remote autonomous vehicle must track a reference signal generated by a remote operator which is communicated to it via a wireless link. In this application, the measurements from on-board sensors are also communicated to remote operator to help operator

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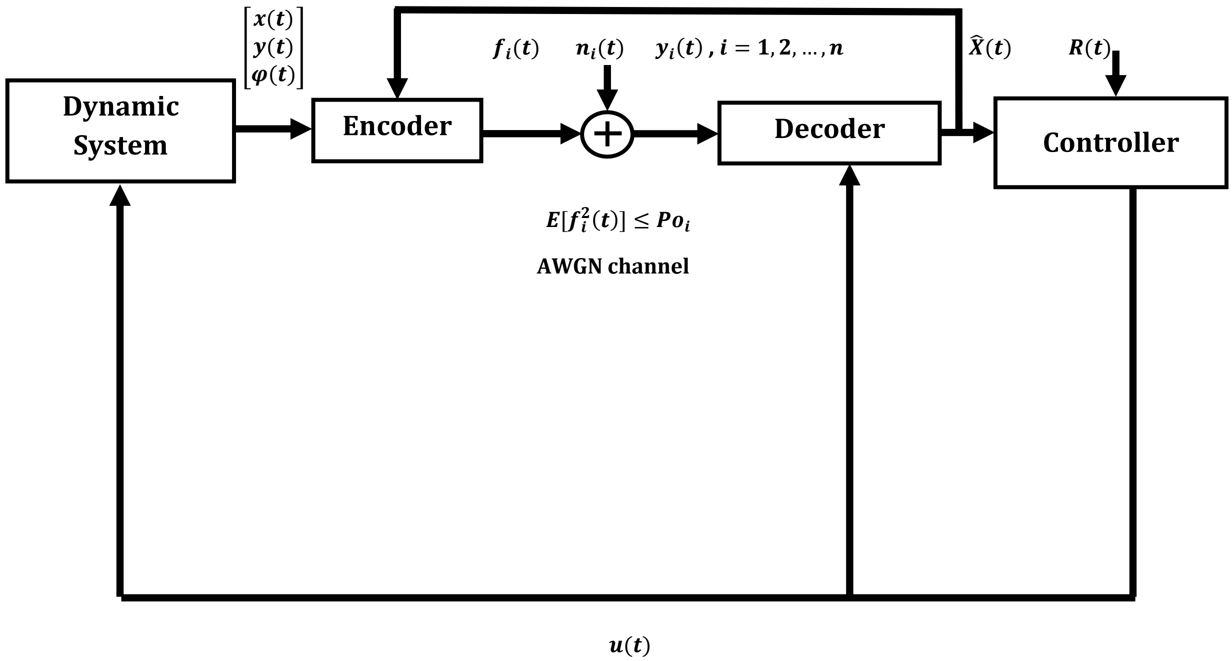

Fig. 1. A dynamic system over MIMO parallel AWGN channel

to generate a desired reference signal. One of the abstract model for wireless communication is Additive White Gaussian Noise (AWGN) channel. This channel is an abstract model for satellite communication, deep space communication and when the line of sight is strong. Therefore, for these situations we deal with the tele-operation of system of Fig. 1. Very often autonomous vehicles are battery powered; and hence, communication from these vehicles to the base station where the operator is located must be done with minimum possible transmission power to increase the on - board battery life time. Therefore, communication from vehicle to remote controller is subject to noise and power constraint. However, as the communication from the base station to remote vehicle can be done with high transmission power, in the block diagram of Fig. 1, the communication link from remote controller to the remote vehicle (dynamic system) can be considered without imperfections and limitations.

The tele-operation of Fig. 1 is an example of networked control systems. In networked control systems we deal with controlling dynamic systems over communication channels subject to imperfections and limitations. Some results addressing basic problems in stability and/or state tracking of dynamic systems over communication channels subject to imperfections and limitations can be found in [1]-[28]. Dynamic systems can be viewed as continuous alphabet information sources with memory. Therefore, many works in the literature (e.g., [1], [12], [13], [18], [19], [22]-[28]) are dedicated to the question of state tracking and/or stability over AWGN channel, which itself is naturally a continuous alphabet channel. [12], [13] addressed the problem of mean square stability and state tracking of linear Gaussian dynamic systems over AWGN channel when noiseless feedback channel is available full time and the communication of control signal from remote controller to system is perfect (see Fig. 1). In [12], the authors presented an optimal control technique for asymptotic bounded mean square stability of partially observed discrete time linear Gaussian systems over AWGN channel. In [13], the authors addressed the continuous time version of the problem addressed in [12]. In [22], the authors considered a framework for discussing control over a communication channel based on Signal-to-Noise Ratio (SNR) constraints and focused particularly on the feedback stabilization of an open loop unstable plant via a channel with a SNR constraint. By examining the simple case of a linear time invariant plant and an AWGN channel, the authors in [22] derived necessary and sufficient conditions on the SNR for feedback stabilization with an LTI controller. In [23], the authors addressed the problem of state tracking of nonlinear systems over AWGN channel. In [25], the authors presented a sub-optimal decentralized control technique for bounded mean square stability of a large scale system with cascaded clusters of sub-systems. Each sub-system is linear and time invariant and both sub-system and its measurement are subject to Gaussian noise. The control signals are exchanged between sub-systems without any imperfections, but the measurements are exchanged via an AWGN communication network. In [27] the author presented a sub-optimal technique for mean square stability of a distributed system with geographically separated Gaussian sub-systems interconnected by a AWGN communication network. In [28], the authors investigated stabilization and performance issues for MIMO LTI networked feedback systems, in which the MIMO communication link is modeled as a parallel noisy AWN channel. The idea of using the describing function for controlling nonlinear dynamic systems over AWGN channel for the first time was presented in [29]. In addition of the AWGN channel, others communication channels, which are in the sharp attention of networked control systems research community, are the real and the packet erasure channels that are abstract models for communication via the Internet and WiFi links. [30] addressed the problems of optimal control and stability of LTI systems over the real erasure channel, which is subject to random packet dropout, where both remote and local controllers operate on the system. [31] also addressed the problem of optimal control of LTI systems by a remote controller over the real erasure channel.

The above literature review reveals that the available results on the stability and state tracking of dynamic systems over AWGN channel are mainly concerned with linear dynamic systems. To the best of our knowledge, control of nonlinear dynamic systems over AWGN channel is limited to [23] and [29], which are not concerned with the tele-operation of autonomous vehicles subject to communication imperfections. In the tele-operation of autonomous vehicles, we deal with state tracking as well as reference tracking of nonlinear systems. The dynamic of autonomous vehicles (autonomous underwater, unmanned aerial and autonomous road vehicles) is described by the six degrees freedom model. However, the vehicle dynamic is handled by an on - board control loop that results in a kinematic unicycle model, which is nonlinear [16]. In addition in deep space tele-operation systems or when the line of sight is strong, AWGN channel is a suitable model for wireless communication. These motivate research on tele-operation of the unicycle model over AWGN channel, which is the subject of this paper.

B. Paper Contributions

Key contributions of this paper compared to the aforementioned earlier literature are summarized as follows:

1) In the field of nonlinear dynamic systems, the describing function is used for building oscillators [32]. However, in this paper, for the first time it is used to stabilize and control a practical nonlinear dynamic system. That is, the unicycle model, which is an abstract model for describing the dynamics of autonomous underwater, unmanned aerial and autonomous road vehicles.

2) To the best of our knowledge, earlier works on controlling dynamic systems over the AWGN channel are mainly concerned with the stability and state tracking of linear dynamic systems, e.g., [1],[12],[13],[18],[19],[22],[24],[25],[27],[28]. Only [24] and [29] addressed the problem of controlling nonlinear dynamic systems over the AWGN channel. Nevertheless, [24] is just concerned with the estimation problem; and [29] addressed the problem of tracking and stability of those systems that have periodic outputs to sinusoidal inputs. Therefore, [29] used a linear dynamic system subject to saturated excitation for computer simulation as this type of non-common dynamic systems is consistent with the theoretical development in [29]. However, by finding a describing function for the unicycle model, which also has periodic outputs to periodic inputs, this paper presents a real application of the theoretical development of [29] by controlling a practical nonlinear dynamic system over the AWGN channel.

C. Paper Organization

The paper is organized as follows. In Section II, the problem formulation is presented. Section III describes the describing function method. Then, in Section IV the theory of mean square reference tracking of linear dynamic systems with multiple real and non-real valued eigenvalues over MIMO AWGN channel is developed. Section V is devoted to the tele-operation of autonomous vehicles. Simulation results for the unicycle model are given in Section VI and the paper is concluded by summarizing the contributions of the paper and direction for future research in Section VII.

II. Problem Formulation

Throughout, certain conventions are used: $E[\cdot]$ denotes the expected value, $var[.]$ the variance, $|\cdot|$ the absolute value and V' the transpose of vector/matrix V. A^{-1} denotes the inverse of a square matrix A and $N(m, n)$ the Gaussian distribution with mean m and covariance n. $\mathbb{R}$ denotes the set of real numbers and I_n the identity matrix with dimension n by n. $trac(A)$ denotes the trace of a square matrix A, $diag\{.\}$ denotes the diagonal matrix, $[A]_{ij}$ denotes the i, jth element of the matrix A and $\underline{0}$ denotes the zero vector/matrix.

This paper is concerned with asymptotic mean square stability and reference tracking of autonomous vehicles over AWGN communication channel, as is shown in the block diagram of Fig. 1. The building blocks of Fig. 1 are described below.

Dynamic System: The dynamic system is the following time - invariant unicycle system [16]:

$$\begin{cases} \dot{x}(t) = v(t)\cos(\phi(t)) \\ \dot{y}(t) = v(t)\sin(\phi(t)) \\ \dot{\phi}(t) = u(t) \end{cases} \tag{1}$$

where $x(t)$, $y(t)$ are the position vector, $\phi(t)$ the heading angle, and the control inputs are the vehicle forward velocity $v(t)$ and the angular velocity $u(t)$. Note that for the remote controller the initial conditions ($x(0)$, $y(0)$, $\phi(0)$) are unknown and has the Gaussian distribution.

Communication Channel: Communication channel between system and controller is a MIMO AWGN channel without interference (parallel) with n inputs and n outputs. The output of the encoder (which will be described shortly) is transmitted through the MIMO channel and a white Gaussian noise vector is added to it (as is shown in Fig. 1), where $N(t) = \begin{bmatrix} n_1(t) & . & . & . & n_n(t) \end{bmatrix}'$ i.i.d. $\sim N(\underline{0}, \tilde{R})$ ($\tilde{R} = diag\{\tilde{r}_1, ..., \tilde{r}_n\}$) is the MIMO channel noise and $n_i(t) \sim N(0, \tilde{r}_i)$ is the additive noise of the ith input - output path of the MIMO parallel AWGN channel. Also, the MIMO parallel AWGN channel is subject to the channel input power constraints Po_i,

$i = 1, 2, ..., n$ as follows: $E[f_i^2(t)] \leq Po_i$, $i = 1, 2, ..., n$, where $f_i(t)$ is the ith element of the encoder output vector $F(t) = \begin{bmatrix} f_1(t) & . & . & . & f_n(t) \end{bmatrix}'$, which is the input of the channel. That is, $Y(t) = F(t) + N(t)$, where $Y(t) = \begin{bmatrix} y_1(t) & . & . & . & y_n(t) \end{bmatrix}'$ is the channel output.

Encoder: The encoder is a bandpass filter cascaded with a matrix gain. This bandpass filter saves only the fundamental frequency of the system outputs and omits the other harmonics which have less information to be sent. For this purpose, a high pass filter with a relatively low cut off frequency (e.g., 0.1 Hz) is used to omit the DC part. This filter is cascaded with a low pass filter with a cut off frequency of $\omega \gg 2\pi \times 0.1$ rad/s. For constructing such a filter we can use the following transfer functions:

$$H_{bp}(s) = H_{hp}(s)H_{lp}(s), \tag{2}$$

$$H_{hp}(s) = \frac{s^2}{s^2 + \frac{\omega_h}{Q}s + \omega_h^2}, \omega_h = 2\pi \times 0.1 \; rad/s, \; Q = damping \; factor = 0.707, \tag{3}$$

$$H_{lp}(s) = \frac{\omega^2}{s^2 + \frac{\omega}{Q}s + \omega^2}. \tag{4}$$

It will be shown in the next section that the nonlinear dynamic system (1) together with the bandpass filter has an approximate linear dynamic system with $n = 7$ states $X(t) = \begin{bmatrix} x_1(t) & . & . & . & x_n(t) \end{bmatrix}'$, in which these states and the matrix gain

$$C(t) = \begin{bmatrix} c_{11}(t) & c_{12}(t) & ... & c_{1n}(t) \\ c_{21}(t) & c_{22}(t) & ... & c_{2n}(t) \\ & & ... & \\ c_{n1}(t) & c_{n2}(t) & ... & c_{nn}(t) \end{bmatrix}$$

form the encoder output as $F(t) = C(t)(X(t) - \hat{X}(t))$, where $\hat{X}(t) = \begin{bmatrix} \hat{x}_1(t) & . & . & . & \hat{x}_n(t) \end{bmatrix}'$ is the estimation of states.

Decoder: Decoder is the minimum mean square estimator or the Kalman filter, also known as Linear Quadratic Estimator (LQE). At each time instant t, the Kalman filter generates an estimation $\hat{X}(t)$ using the channel output $Y(t)$.

Controller: Controller is a certainty equivalent controller of the following form: $u(t) = -l(t)\hat{X}(t) + \nu(t)$.

The objective of this paper is to find the matrix gain $C(t)$ and $u(t)$ to force the positions $x(t)$ and $y(t)$ and the heading angle $\phi(t)$ follow the desired paths.

III. Implementation of the Describing Function Method

In this section, we first present the idea of the describing function to obtain the approximate linear dynamic system from a nonlinear time-invariant dynamic system. Then, we obtain the describing function for the nonlinear dynamic system of (1).

A. The Describing Function

This subsection is borrowed from [29]. For all nonlinear time - invariant dynamic systems that respond periodically to sinusoidal inputs, we can find an approximate linear dynamic system, as defined below [32]:

Consider a SISO nonlinear time - invariant dynamic system with periodic outputs in response to periodic inputs. Suppose that this nonlinear dynamic system is excited by the following input: $u(t) = \gamma \cos(\omega t)$, where $\gamma > 0$ is large enough to excite all modes of the nonlinear system. Then, as the output is a periodic signal, it has a Fourier series representation that includes all harmonics of the input with frequency of ω. That is,

$$\begin{aligned} y(t) &= y_d + \sum_{i=1}^{\infty} (a_i \sin(i\omega t) + b_i \cos(i\omega t)), \\ y_d &= \frac{\omega}{4\pi} \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} y(t) dt, \\ a_i &= \frac{\omega}{2\pi} \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} (y(t) \sin(i\omega t)) dt \\ b_i &= \frac{\omega}{2\pi} \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} (y(t) \cos(i\omega t)) dt. \end{aligned} \tag{5}$$

Now, if this nonlinear system is cascaded with a bandpass filter with high cut-off frequency of ω, then we have a periodic output at the end of the filter which consists of only the first harmonic with the frequency of ω, and all other harmonics are eliminated. In other words, the output of the bandpass filter is the following:

$$y_f(t) = a_1 \sin(\omega t) + b_1 \cos(\omega t). \tag{6}$$

Having that, we call the nonlinear dynamic system that is cascaded with the bandpass filter with the high cut off frequency of ω, a quasi linear system [32]. Because, we can find a linear dynamic system with the input $u(t) = \gamma \cos(\omega t)$ and the output $y_f(t)$ with the following transfer function:

$$H(j\omega) = |H(j\omega)| \angle H(j\omega). \tag{7}$$

$$|H(j\omega)| = \frac{\sqrt{a_1^2 + b_1^2}}{\gamma}. \tag{8}$$

$$\angle H(j\omega) = -\arctan(\frac{a_1}{b_1})\ rad. \tag{9}$$

This means that the nonlinear dynamic system can be represented by a linear dynamic system with the above transfer function called the describing function of the nonlinear dynamic system. Note that the describing function represents the transfer function of the approximate linear dynamic system and the transfer function represents the response of the dynamic system to the input signal when the initial conditions are set to be zero. Hence, the describing function is obtained for zero initial conditions.

B. The Approximate Linear System for the Unicycle Model

For a given fixed forward velocity $v(t)$, the nonlinear dynamic system (1) has periodic outputs to periodic inputs $u(t) = \gamma cos(\omega t)$. Hence, in the block diagram of Fig. 1 as this dynamic is cascaded with the bandpass filter, it has an approximate linear system description. To obtain this describing function, we first assume that the input is $u(t) = \gamma \cos(\omega t)$; and $\dot{x}(t)$ and $\dot{y}(t)$ are the outputs of the system (1); and hence, the inputs of the bandpass filter. Therefore, we obtain $H_{\dot{x}}(s) = \frac{f_1}{f_2 s^2 + f_3 s + f_4}$ and $H_{\dot{y}}(s) = \frac{c_1}{c_2 s^2 + c_3 2 s + c_4}$ transfer functions as describing functions from the input to the outputs $\dot{x}(t)$ and $\dot{y}(t)$, respectively (f_is and c_is are real coefficients). Then, obviously for obtaining the describing functions from input $u(t)$ to outputs $x(t)$ and $y(t)$, we must multiply $\frac{1}{s}$ to these transfer functions. Note that as is clear from Fig. 1, the inputs to the bandpass filter are $x(t)$ and $y(t)$. However; we can assume that these inputs are obtained by integration of $\dot{x}(t)$ and $\dot{y}(t)$; and thus, for the simplicity in obtaining the describing functions, by moving this integration operator after the transfer function of the bandpass filter, we can assume that the outputs of the nonlinear system and therefore the inputs to the filter are $\dot{x}(t)$ and $\dot{y}(t)$. Also, note that from (1) it follows that the transfer function between input $u(t)$ and output $\phi(t)$ is $H_\phi(s) = \frac{1}{s}$. That is, the relation between $u(t)$ and $\phi(t)$ is linear .

The equivalent state space representation of the approximate linear system has seven states: $X(t) = [x_1(t)\ x_2(t)\ .\ .\ .\ x_7(t)]'$ where correspond to $d_1\ddot{x}(t) + d_2\dot{x}(t) + x(t)$, $d_3\ddot{x}(t) + d_4\dot{x}(t)$, $d_5\ddot{x}(t) + d_6\dot{x}(t)$, $d_7\ddot{y}(t) + d_8\dot{y}(t) + y(t)$, $d_9\ddot{y}(t) + d_{10}\dot{y}(t)$, $d_{11}\ddot{y}(t) + d_{12}\dot{y}(t)$ and $\phi(t)$, respectively (d_is are real coefficients). The input of this representation is $u(t)$ and the output vector is

$[x(t)\ y(t)\ \phi(t)]'$ with the system matrices $A = \begin{bmatrix} A_x & \underline{0} & \underline{0} \\ \underline{0} & A_y & \underline{0} \\ \underline{0} & \underline{0} & A_\phi \end{bmatrix}$, $B = [B_x \quad B_y \quad B_\phi]'$ and $C = \begin{bmatrix} C_x & \underline{0} & \underline{0} \\ \underline{0} & C_y & \underline{0} \\ \underline{0} & \underline{0} & C_\phi \end{bmatrix}$, where $A_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & e_1 & e_2 \\ 0 & -e_2 & e_1 \end{bmatrix}$, $A_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & e_5 & e_6 \\ 0 & -e_6 & e_5 \end{bmatrix}$, $A_\phi = 0$, $B_x = [e_9\ e_{10}\ e_{11}]$, $B_y = [e_{12}\ e_{13}\ e_{14}]$, $B_\phi = 1$, $C_x = [e_{15}\ e_{16}\ 0]$, $C_y = [e_{17}\ e_{18}\ 0]$ and $C_\phi = 1$. Hence, we need a MIMO parallel AWGN channel with 7 inputs and 7 outputs for transmitting $x_1(t)$ to $x_7(t)$. Note that the equivalent state space representation of the approximate linear system is in the real Jordan form.

IV. Reference Tracking of Linear Systems with Multiple Real and Non-real Valued Eigenvalues over AWGN Channel

Now, we extend the results of [13] to account for reference tracking and hence stability of linear dynamic systems with multiple real and non-real valued eigenvalues over MIMO parallel AWGN channel. In the next section, by applying these extended results on the describing functions associated with the nonlinear system (1), we address the reference tracking problem of tele-operation of autonomous vehicles, as is shown in the block diagram of Fig. 1.

Suppose that the dynamic system in Fig. 1 is linear with n states. Hence, the system that is seen by the remote controller is as follows:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t),\ X(0) = \xi \\ Y(t) = C(t)(X(t) - \hat{X}(t)) + N(t) \end{cases} \tag{10}$$

where $X(0)$ is the initial state, which is known for the encoder but unknown for the remote controller, that is for the remote controller the exact value of ξ is unknown and hence $\xi \sim N(X_0, Q_0)$ (Q_0 is diagonal) is treated as the Gaussian random variable with known mean and variance. Note that $N(t) = \begin{bmatrix} n_1(t) & . & . & . & n_n(t) \end{bmatrix}'$ $i.i.d. \sim N(\underline{0}, \tilde{R})$ ($\tilde{R} = diag\{\tilde{r}_1, ..., \tilde{r}_n\}$) is the additive noise of the MIMO channel ($n_i(t) \sim N(0, \tilde{r}_i)$ is the additive noise of the ith path of the MIMO parallel AWGN channel) which can be treated as the measurement noise provided the channel input power constraint is met. The objective here is to achieve mean square asymptotic reference tracking for the linear system (10) with n states and n outputs over MIMO parallel AWGN channel.

Throughout, it is assumed that the system matrix A has real eigenvalues, real multiple eigenvalues and distinct complex conjugate eigenvalues as the system matrix A that corresponds to the describing function of the nonlinear system (1) includes these types of eigenvalues. As it was shown in the previous section, for the unicycle model, the system matrix A of the approximate linear system is in the real Jordan form. Therefore, throughout this section it is assumed that the system (10) can be decomposed to several decoupled sub-systems; and hence, for each sub-system, an encoder and a decoder are designed separately. Note that the Jordan block associated with a real eigenvalue $\lambda_i(A)$ with multiplicity 2 is the following matrix $\begin{bmatrix} \lambda_i(A) & 1 \\ 0 & \lambda_i(A) \end{bmatrix}$ and the Jordan block associated with the complex conjugate pair of eigenvalues $\lambda_i(A) = \sigma \pm \sqrt{-1}w$ $(w \neq 0)$ is $\begin{bmatrix} \sigma & w \\ -w & \sigma \end{bmatrix}$.

A. *Mean square asymptotic state tracking*

In this section, it is assumed that each of the Jordan block is at most a 2 by 2 matrix. Then, for all three possible cases: a) $A = \begin{bmatrix} \sigma & w \\ -w & \sigma \end{bmatrix}$, $\sigma, w \in \mathbb{R}, w \neq 0$, b) $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, $a \in \mathbb{R}$, c) $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, $a_1, a_2 \in \mathbb{R}, a_1 \neq a_2$; we find the matrix gain $C(t)$ for mean square asymptotic state tracking of system states at the decoder.

1) Sub-systems with complex conjugate eigenvalues: Suppose that the system matrix A in the linear system of (10) has the following form

$$A = \begin{bmatrix} \sigma & w \\ -w & \sigma \end{bmatrix}, \sigma, w \in \mathbb{R}, w \neq 0. \tag{11}$$

Then, we have the following proposition for mean square asymptotic state tracking of system states at the decoder:

Proposition 4.1: Consider the block diagram of Fig. 1 described by the linear system (10) with the system matrix $A = \begin{bmatrix} \sigma & w \\ -w & \sigma \end{bmatrix}, \sigma, w \in \mathbb{R}, w \neq 0$. Suppose that $Po_1 > 2\sigma$ and $Po_2 > 2\sigma$, $[Q_0]_{12} = 0$, $[Q_0]_{11} = [Q_0]_{22}$ and $\tilde{r}_1 = \tilde{r}_2$. Let $\gamma_1 = \min(Po_1, Po_2)$. Then, by choosing the encoder matrix gain as $C(t) = \begin{bmatrix} \sqrt{\frac{\gamma_1 \tilde{r}_1}{p_{11}(t)}} & 0 \\ 0 & \sqrt{\frac{\gamma_1 \tilde{r}_2}{p_{22}(t)}} \end{bmatrix}$, and the following decoder

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \tag{12}$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1}, \;\; \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\} \tag{13}$$

$$\dot{P}(t) = A'P(t) + P(t)A - P(t)C'(t)\tilde{R}^{-1}C(t)P(t),\ P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) \\ p_{12}(t) & p_{22}(t) \end{bmatrix} \geq 0; \quad (14)$$

we have mean square asymptotic state tracking of system states at the decoder.

Proof: The encoder matrix gain has the following general form $C(t) = \begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix}$; and subsequently the decoder can be extracted as follows (for the simplicity of presentation the dependency to the time index, t, is dropped):

$$\begin{cases} \dot{p}_{11} = 2\sigma p_{11} - 2wp_{12} - (2p_{11}p_{12}c_{11}c_{12}\tilde{r}_1^{-1} + 2p_{11}p_{12}c_{21}c_{22}\tilde{r}_2^{-1} + p_{11}^2c_{11}^2\tilde{r}_1^{-1} + p_{12}^2c_{12}^2\tilde{r}_1^{-1} \\ \qquad + p_{11}^2c_{21}^2\tilde{r}_2^{-1} + p_{12}^2c_{22}^2\tilde{r}_2^{-1}) \\ \dot{p_{22}} = 2\sigma p_{22} + 2wp_{12} - (2p_{22}p_{12}c_{22}c_{21}\tilde{r}_2^{-1} + 2p_{22}p_{12}c_{21}c_{11}\tilde{r}_1^{-1} + p_{22}^2c_{22}^2\tilde{r}_2^{-1} + p_{12}^2c_{21}^2\tilde{r}_2^{-1} \\ \qquad + p_{22}^2c_{12}^2\tilde{r}_1^{-1} + p_{12}^2c_{11}^2\tilde{r}_1^{-1}) \\ \dot{p_{12}} = 2\sigma p_{12} - wp_{22} + wp_{11} - \tilde{r}_1^{-1}(p_{11}p_{12}c_{11}^2 + p_{12}^2c_{11}c_{12} + p_{11}p_{22}c_{11}c_{12} + p_{12}p_{22}c_{12}^2) \\ \qquad -\tilde{r}_2^{-1}(p_{11}p_{12}c_{21}^2 + p_{12}^2c_{22}c_{21} + p_{11}p_{22}c_{22}c_{21} + p_{12}p_{22}c_{22}^2) \end{cases} \quad (15)$$

Now, by substituting $c_{11}(t) = \sqrt{\frac{\gamma_1\tilde{r}_1}{p_{11}(t)}}$, $c_{22}(t) = \sqrt{\frac{\gamma_1\tilde{r}_2}{p_{22}(t)}}$ and $c_{12}(t) = c_{21}(t) = 0$, we have $\dot{p}_{12}(t) = 0$ ($\Rightarrow p_{12}(t) = p_{12}(0) = [Q_0]_{12} = 0$) and

$$\dot{p}_{11}(t) = (2\sigma - \gamma_1)p_{11}(t) \Rightarrow p_{11}(t) = e^{-(\gamma_1-2\sigma)t}p_{11}(0). \quad (16)$$

$$\dot{p}_{22}(t) = (2\sigma - \gamma_1)p_{22}(t) \Rightarrow p_{22}(t) = e^{-(\gamma_1-2\sigma)t}p_{22}(0). \quad (17)$$

Thus, by the above selection, we have

$$P(t) = \begin{bmatrix} e^{-(\gamma_1-2\sigma)t}p_{11}(0) & 0 \\ 0 & e^{-(\gamma_1-2\sigma)t}p_{22}(0) \end{bmatrix} \quad (18)$$

when the decoder description is as follows:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \quad K(t) = P(t)C^{'}(t)\tilde{R}^{-1}. \quad (19)$$

Now, as we assumed that $\gamma_1 > 2\sigma$, we have $\lim_{t\to\infty} E[(x_1(t) - \hat{x}_1(t))^2] = \lim_{t\to\infty} p_{11}(t) = \lim_{t\to\infty} e^{-(\gamma_1-2\sigma)t}p_{11}(0) = 0$. Similarly, we have $\lim_{t\to\infty} E[(x_2(t)-\hat{x}_2(t))^2] = \lim_{t\to\infty} p_{22}(t) = 0$. Hence, $P(t) \to \bar{P} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. This completes the proof.

2) Sub-systems with real multiple eigenvalues: Suppose that the system matrix A in the linear system of (10) has the following form

$$A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, a \in \mathbb{R}. \tag{20}$$

Then, we have the following proposition for mean square asymptotic state tracking of system states at the decoder:

Proposition 4.2: Consider the block diagram of Fig. 1 described by the linear system (10) with the system matrix $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, $a \in \mathbb{R}$. Suppose that $Po_1 > 2a$ and $Po_2 > 2a$, $[Q_0]_{12} = 0$, $[Q_0]_{11} = [Q_0]_{22}$ and $\tilde{r}_1 = \tilde{r}_2$. Then, by choosing the encoder matrix gain as $C(t) = \begin{bmatrix} \sqrt{\frac{\tilde{r}_1}{2\delta p_{11}(t)}} & \sqrt{\frac{\delta\tilde{r}_1}{2p_{22}(t)}} \\ \sqrt{\frac{\delta\tilde{r}_2}{2p_{11}(t)}} & \sqrt{\frac{\tilde{r}_2}{2\delta p_{22}(t)}} \end{bmatrix}$, where $\delta = \gamma_1 - \sqrt{\gamma_1^2 - 1}$ and $\gamma_1 = \min(Po_1, Po_2)$, and the following decoder

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \tag{21}$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1}, \ \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\} \tag{22}$$

$$\dot{P}(t) = A'P(t) + P(t)A - P(t)C'(t)\tilde{R}^{-1}C(t)P(t), \ P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) \\ p_{12}(t) & p_{22}(t) \end{bmatrix} \geq 0; \tag{23}$$

we have mean square asymptotic state tracking of system states at the decoder.

Proof: The encoder matrix gain has the following general form $C(t) = \begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix}$; and subsequently the decoder can be extracted as follows (for the simplicity of presentation the dependency to the time index, t, is dropped):

$$\begin{cases} \dot{p}_{11} = 2ap_{11} - (2p_{11}p_{12}c_{11}c_{12}\tilde{r}_1^{-1} + 2p_{11}p_{12}c_{21}c_{22}\tilde{r}_2^{-1} + p_{11}^2c_{11}^2\tilde{r}_1^{-1} + p_{12}^2c_{12}^2\tilde{r}_1^{-1} \\ \qquad + p_{11}^2c_{21}^2\tilde{r}_2^{-1} + p_{12}^2c_{22}^2\tilde{r}_2^{-1}) \\ \dot{p}_{22} = 2ap_{22} + 2p_{12} - (2p_{22}p_{12}c_{22}c_{21}\tilde{r}_2^{-1} + 2p_{22}p_{12}c_{21}c_{11}\tilde{r}_1^{-1} + p_{22}^2c_{22}^2\tilde{r}_2^{-1} + p_{12}^2c_{21}^2\tilde{r}_2^{-1} \\ \qquad + p_{22}^2c_{12}^2\tilde{r}_1^{-1} + p_{12}^2c_{11}^2\tilde{r}_1^{-1}) \\ \dot{p}_{12} = 2ap_{12} + p_{11} - \tilde{r}_1^{-1}(p_{11}p_{12}c_{11}^2 + p_{12}^2c_{11}c_{12} + p_{11}p_{22}c_{11}c_{12} + p_{12}p_{22}c_{12}^2) \\ \qquad - \tilde{r}_2^{-1}(p_{11}p_{12}c_{21}^2 + p_{12}^2c_{22}c_{21} + p_{11}p_{22}c_{22}c_{21} + p_{12}p_{22}c_{22}^2) \end{cases} \tag{24}$$

Now, by substituting $c_{11}(t) = \sqrt{\frac{\tilde{r}_1}{2\delta p_{11}(t)}}$, $c_{12}(t) = \sqrt{\frac{\delta\tilde{r}_1}{2p_{22}(t)}}$, $c_{21}(t) = \sqrt{\frac{\delta\tilde{r}_2}{2p_{11}(t)}}$ and $c_{22}(t) = \sqrt{\frac{\tilde{r}_2}{2\delta p_{22}(t)}}$, we have $\dot{p}_{12}(t) = 0$ ($\Rightarrow p_{12}(t) = p_{12}(0) = [Q_0]_{12} = 0$) and

$$\dot{p}_{11}(t) = (2a - (\frac{1}{2\delta} + \frac{\delta}{2}))p_{11}(t) \Rightarrow \dot{p}_{11}(t) = (2a - \frac{1+\delta^2}{2\delta})p_{11}(t) \Rightarrow \tag{25}$$

$$\dot{p}_{11}(t) = (2a - \gamma_1)p_{11}(t) \Rightarrow p_{11}(t) = e^{-(\gamma_1 - 2a)t}p_{11}(0). \tag{26}$$

$$\dot{p}_{22}(t) = (2a - (\frac{1}{2\delta} + \frac{\delta}{2}))p_{22}(t) \Rightarrow \dot{p}_{22}(t) = (2a - \frac{1+\delta^2}{2\delta})p_{22}(t) \Rightarrow \tag{27}$$

$$\dot{p}_{22}(t) = (2a - \gamma_1)p_{22}(t) \Rightarrow p_{22}(t) = e^{-(\gamma_1 - 2a)t}p_{22}(0). \tag{28}$$

Thus, by the above selection, we have

$$P(t) = \begin{bmatrix} e^{-(\gamma_1-2a)t}p_{11}(0) & 0 \\ 0 & e^{-(\gamma_1-2a)t}p_{22}(0) \end{bmatrix} \tag{29}$$

when the decoder description is as follows:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \quad K(t) = P(t)C^{'}(t)\tilde{R}^{-1}. \tag{30}$$

Now, as we assumed that $Po_1 > 2a$ and $Po_2 > 2a$, we have $\lim_{t\to\infty} E[(x_1(t) - \hat{x}_1(t))^2] = \lim_{t\to\infty} p_{11}(t) = \lim_{t\to\infty} e^{-(\gamma_1-2a)t}p_{11}(0) = 0$. Similarly, we have $\lim_{t\to\infty} E[(x_2(t) - \hat{x}_2(t))^2] = \lim_{t\to\infty} p_{22}(t) = 0$. Hence, $P(t) \to \bar{P} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. This completes the proof.

3) Sub-systems with real distinct eigenvalues: Suppose that the system matrix A in the linear system of (10) has the following form

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, a_1, a_2 \in \mathbb{R},\ a_1 \neq a_2. \tag{31}$$

Then, we have the following proposition for mean square asymptotic state tracking of system states at the decoder:

Proposition 4.3: Consider the block diagram of Fig. 1 described by the linear system (10) with the system matrix $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, $a_1, a_2 \in \mathbb{R}, a_1 \neq a_2$. Suppose that $[Q_0]_{12} = 0$, $Po_1 > 2a_1$ and $Po_2 > 2a_2$ (Po_1 and Po_2 are the channel input power constraint). Then, by choosing the encoder matrix gain as $C(t) = \begin{bmatrix} \sqrt{\frac{Po_1\tilde{r}_1}{p_{11}(t)}} & 0 \\ 0 & \sqrt{\frac{Po_2\tilde{r}_2}{p_{22}(t)}} \end{bmatrix}$, and a decoder with the following description

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \tag{32}$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1}, \ \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\} \tag{33}$$

$$\dot{P}(t) = A'P(t) + P(t)A - P(t)C'(t)\tilde{R}^{-1}C(t)P(t), \ P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) \\ p_{12}(t) & p_{22}(t) \end{bmatrix} \geq 0; \tag{34}$$

we have mean square asymptotic state tracking of system states at the decoder.

Proof: The encoder matrix gain has the following general form $C(t) = \begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix}$; and subsequently, the matrix $P(t)$, which represents the covariance matrix of the decoding error can be extracted as follows (for the simplicity of presentation the dependency to the time index t is dropped):

$$\begin{cases} \dot{p}_{11} = 2a_1p_{11} - (2p_{11}p_{12}c_{11}c_{12}\tilde{r}_1^{-1} + 2p_{11}p_{12}c_{21}c_{22}\tilde{r}_2^{-1} + p_{11}^2c_{11}^2\tilde{r}_1^{-1} + p_{12}^2c_{12}^2\tilde{r}_1^{-1} \\ \qquad + p_{11}^2c_{21}^2\tilde{r}_2^{-1} + p_{12}^2c_{22}^2\tilde{r}_2^{-1}) \\ \dot{p_{22}} = 2a_2p_{22} - (2p_{22}p_{12}c_{22}c_{21}\tilde{r}_2^{-1} + 2p_{22}p_{12}c_{21}c_{11}\tilde{r}_1^{-1} + p_{22}^2c_{22}^2\tilde{r}_2^{-1} + p_{12}^2c_{21}^2\tilde{r}_2^{-1} \\ \qquad + p_{22}^2c_{12}^2\tilde{r}_1^{-1} + p_{12}^2c_{11}^2\tilde{r}_1^{-1}) \\ \dot{p_{12}} = a_1p_{12} + a_2p_{12} - \tilde{r}_1^{-1}(p_{11}p_{12}c_{11}^2 + p_{12}^2c_{11}c_{12} + p_{11}p_{22}c_{11}c_{12} + p_{12}p_{22}c_{12}^2) \\ \qquad - \tilde{r}_2^{-1}(p_{11}p_{12}c_{21}^2 + p_{12}^2c_{22}c_{21} + p_{11}p_{22}c_{22}c_{21} + p_{12}p_{22}c_{22}^2) \end{cases} \tag{35}$$

Now, by substituting $c_{11}(t) = \sqrt{\frac{Po_1\tilde{r}_1}{p_{11}(t)}}$, $c_{22}(t) = \sqrt{\frac{Po_2\tilde{r}_2}{p_{22}(t)}}$ and $c_{12}(t) = c_{21}(t) = 0$, we have $\dot{p}_{12}(t) = 0$ and $p_{12}(t) = p_{12}(0) = [Q_0]_{12} = 0$. Subsequently, we have

$$\dot{p}_{11}(t) = (2a_1 - Po_1)p_{11}(t) \Rightarrow p_{11}(t) = e^{-(Po_1 - 2a_1)t}p_{11}(0). \tag{36}$$

$$\dot{p}_{22}(t) = (2a_2 - Po_2)p_{22}(t) \Rightarrow p_{22}(t) = e^{-(Po_2 - 2a_2)t}p_{22}(0). \tag{37}$$

Thus, by the above selection, we have

$$P(t) = \begin{bmatrix} e^{-(Po_1 - 2a_1)t}p_{11}(0) & 0 \\ 0 & e^{-(Po_2 - 2a_2)t}p_{22}(0) \end{bmatrix} \tag{38}$$

when the decoder description is as follows:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0, \quad K(t) = P(t)C^{'}(t)\tilde{R}^{-1}. \tag{39}$$

Now, as we assumed that $Po_1 > 2a_1$ and $Po_2 > 2a_2$, we have $\lim_{t\to\infty} E[(x_1(t) - \hat{x}_1(t))^2] = \lim_{t\to\infty} p_{11}(t) = \lim_{t\to\infty} e^{-(Po_1-2a_1)t}p_{11}(0) = 0$. Similarly, we have $\lim_{t\to\infty} E[(x_2(t)-\hat{x}_2(t))^2] = \lim_{t\to\infty} p_{22}(t) = 0$. Hence, $P(t) \to \bar{P} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. This completes the proof.

B. Asymptotic Reference Tracking

Now, to obtain the control signal $u(t)$ for tracking the reference signal $R(t)$, we consider the following cost functional

$$J = \lim_{t_1\to\infty} \frac{1}{t_1}\int_0^{t_1} E[[X(t)-R(t)]'Q[X(t)-R(t)] + ru^2(t)]dt,\ \ Q = Q' \geq 0, r > 0. \tag{40}$$

Subsequently, the control signal $u(t)$ is obtained by minimizing the above cost functional subject to the dynamic system (10). From the classical LQG results [33] it follows that $u(t) = -l(t)\hat{X}(t) + \nu(t)$, where

$$l(t) = r^{-1}B'\tilde{P}(t) \tag{41}$$

$$\nu(t) = -r^{-1}B'S(t) \tag{42}$$

and $\tilde{P}(t)$ and $S(t)$ are the solutions of the following equations:

$$\dot{\tilde{P}}(t) = -\tilde{P}(t)A - A'\tilde{P}(t) - Q + \tilde{P}(t)Br^{-1}B'\tilde{P}(t) \tag{43}$$

$$\dot{S}(t) = -[A' - \tilde{P}(t)Br^{-1}B']S(t) + QR(t). \tag{44}$$

Under the assumption that the pair (A, B) is stabilizable and the pair(I_n, A) is detectable, we have $\tilde{P}(t) \to \bar{\tilde{P}}$ [33], where $\bar{\tilde{P}}$ is the unique symmetric non-negative definite stabilizing solution of the corresponding Algebraic Riccati equation; and hence, $J \to \bar{J} = 0$. This indicates that $E[X(t) - R(t)]'Q[X(t) - R(t)] \to 0$; and hence, $E[(x_i(t) - r_i(t))^2] \to 0$, $i = 1, 2, ..., n$, ($R(t) = \begin{bmatrix} r_1(t) & . & . & . & r_n(t)\end{bmatrix}'$).

V. Reference Tracking of the Tele-operation of Autonomous Vehicles

In this section, we apply the results of previous sections to address the reference tracking problem of the tele-operation system described by the block diagram of Fig. 1. In this block diagram the nonlinear dynamics of the miniature drones, autonomous road vehicles and autonomous underwater vehicles is described by the unicycle model of (1). In this tele-operation system, the initial conditions vector $\begin{bmatrix} x(0) & y(0) & \phi(0)\end{bmatrix}'$ is unknown for the remote controller and has the Gaussian distribution with diagonal covariance matrix. Both positions $x(t)$ and $y(t)$ must be controlled by one input $u(t)$ as it is assumed that the forward velocity $v(t)$ is fixed. In the following $r_\phi(t)$, $r_x(t)$ and $r_y(t)$ are the reference signals for $\phi(t)$, $x(t)$ and $y(t)$, respectively ($R(t) = [r_x(t)\ r_y(t)\ r_\phi(t)]'$); and $\hat{x}(t)$, $\hat{y}(t)$ and $\hat{\phi}(t)$ are mean square estimation of $x(t)$, $y(t)$ and $\phi(t)$ at the decoder, respectively, which are available for the remote controller. Now, from Fig. 2 it follows that to control both $x(t)$ and $y(t)$ by one input $u(t)$, we must have the reference

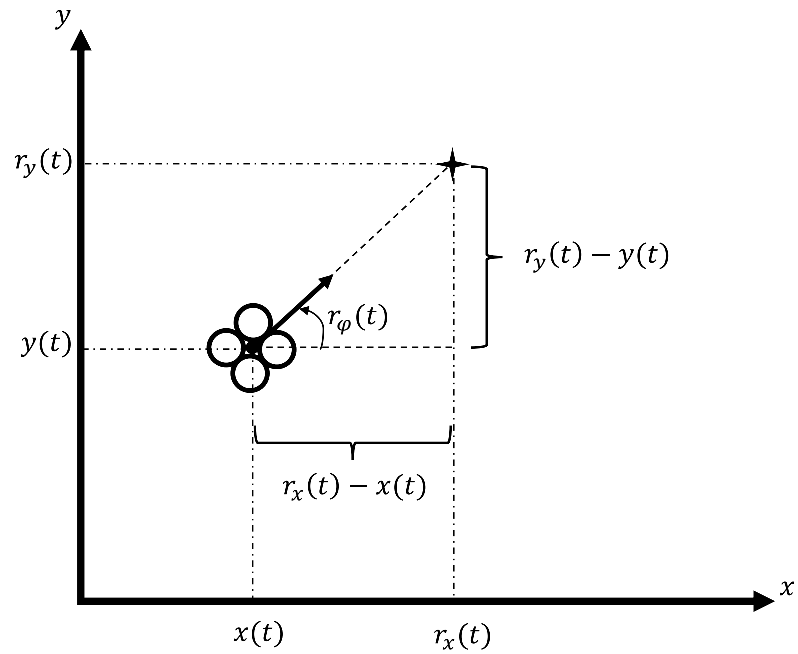

Fig. 2. An autonomous vehicle with current positions $x(t)$ and $y(t)$ moving towards the desired positions $r_x(t)$ and $r_y(t)$

signal $r_\phi(t) = \arctan(\frac{r_y(t)-\hat{y}(t)}{r_x(t)-\hat{x}(t)})$ [34], which forces $x(t)$ and $y(t)$ to follow $r_x(t)$ and $r_y(t)$, respectively; when $\phi(t)$ tracks $r_\phi(t)$ and $\hat{x}(t) \to x(t)$, $\hat{y}(t) \to y(t)$.

To achieve this goal, we notice that the equivalent state space representation of the approximate linear system of the nonlinear unicycle model cascaded with the bandpass filter, has seven states: $X(t) = [x_1(t)\ x_2(t)\ \ldots\ x_7(t)]'$; and hence, we need a MIMO parallel AWGN channel with 7 inputs and 7 outputs for transmitting $x_1(t)$ to $x_7(t)$. Now, we use the encoder and decoder proposed in the previous section with matrices derived from A, B and C as given in Section III. Since the system matrix A is in the real Jordan form, the encoder and decoder proposed in Section IV can be used for different 2 by 2 sub-systems for transmitting $[x_2(t)\ x_3(t)]'$ and $[x_5(t)\ x_6(t)]'$ and their reconstruction at the decoder; and also for the scalar sub-systems for transmitting $x_1(t)$, $x_4(t)$ and $x_7(t)$ and their reconstruction. Then, using the controller $u(t) = -l(t)\hat{\phi}(t) + \nu(t)$ with gains computed by (41) to (44) with $A = A_\phi = 0$, $B = B_\phi = 1$ and $R(t) = r_\phi(t)$, we have mean square asymptotic tracking of the reference signal $r_\phi(t)$ by $\phi(t)$; and hence, the desired reference tracking. Note that at the decoder $\hat{x}(t)$, $\hat{y}(t)$ and $\hat{\phi}(t)$ are obtained as follows: $[\hat{x}(t)\ \hat{y}(t)\ \hat{\phi}(t)]' = C\hat{X}(t)$, which converge to $[x(t)\ y(t)\ \phi(t)]' = CX(t)$, as $\hat{X}(t) \to X(t)$, in mean square sense.

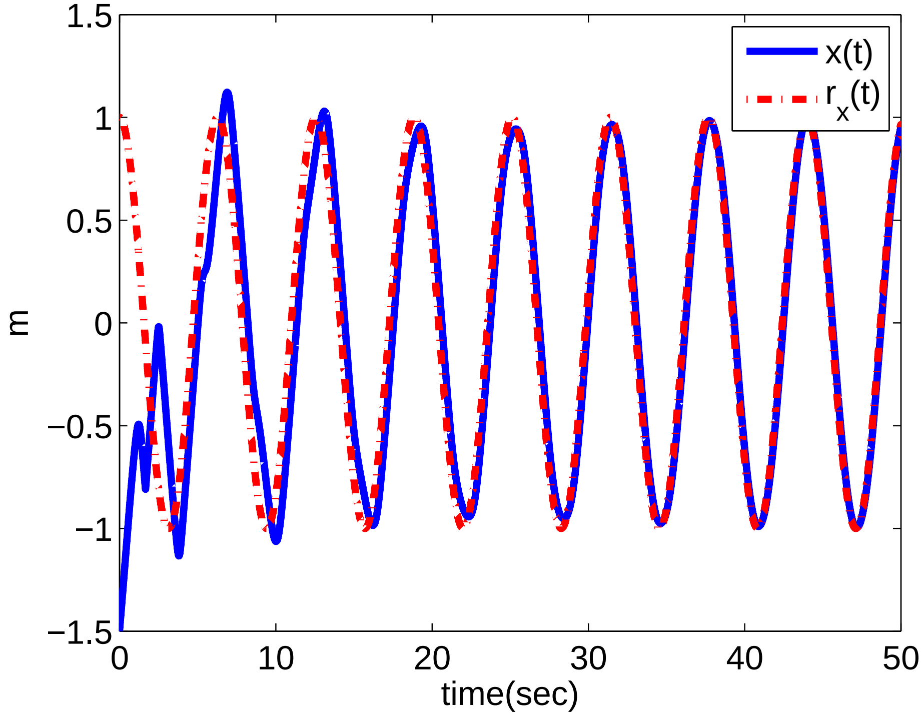

Fig. 3. $x(t)$ and $r_x(t)$

VI. Simulations Results

For the purpose of illustration, consider the block diagram of Fig. 1 with the nonlinear dynamic of (1). For the remote controller, the initial conditions $(x(0), y(0), \phi(0))$ are unknown and has the following description $[x(t)\ y(t)\ \phi(t)]' \sim N(\underline{0}, 3I_3)$. The autonomous vehicle must track a circle with the center located at (x_r, y_r) and the radius of ρ with the angular velocity of ω_r. Therefore, $[x(t)\ y(t)\ \phi(t)]'$ must track the reference signal $[r_x(t)\ r_y(t)\ r_\phi(t)]'$, where $r_x(t) = x_r + \rho\cos(\omega_r t)$, $r_y(t) = y_r + \rho\sin(\omega_r t)$ and $r_\phi(t) = \arctan(\frac{r_y(t)-\hat{y}(t)}{r_x(t)-\hat{x}(t)})$. Note that as $\dot{r}_x(t) = \rho\omega_r\sin(-\omega_r t)$ and $\dot{r}_y(t) = \rho\omega_r\cos(-\omega_r t)$, for the simplicity of design, we choose the forward velocity constant and equals to $v(t) = \rho\omega_r\ m/s$.

To obtain the describing function for a given ω, e.g., $\omega = 1\ rad/s$; we apply the inputs $v(t) = \rho\omega_r\ m/s$ and $u(t) = \eta\omega_r\cos(\omega t)\ rad/s$ ($\eta \geq 1$ is a gain to excite all modes of the nonlinear system) to the system (1). Then, from this input and the corresponding outputs of the bandpass filter, the describing functions for $\eta = 3$, $\omega_r = 1\ rad/s$ and $\rho = 1$ are computed as $H_x(s) = \frac{1}{s(s^2+0.2s+8.5)}$ and $H_y(s) = \frac{-1}{s(s^2+0.2s+8.5)}$, respectively. The equivalent linear state space representation of this system has seven states: $X(t) = [x_1(t)\ x_2(t)\ .\ .\ .\ x_7(t)]'$ where

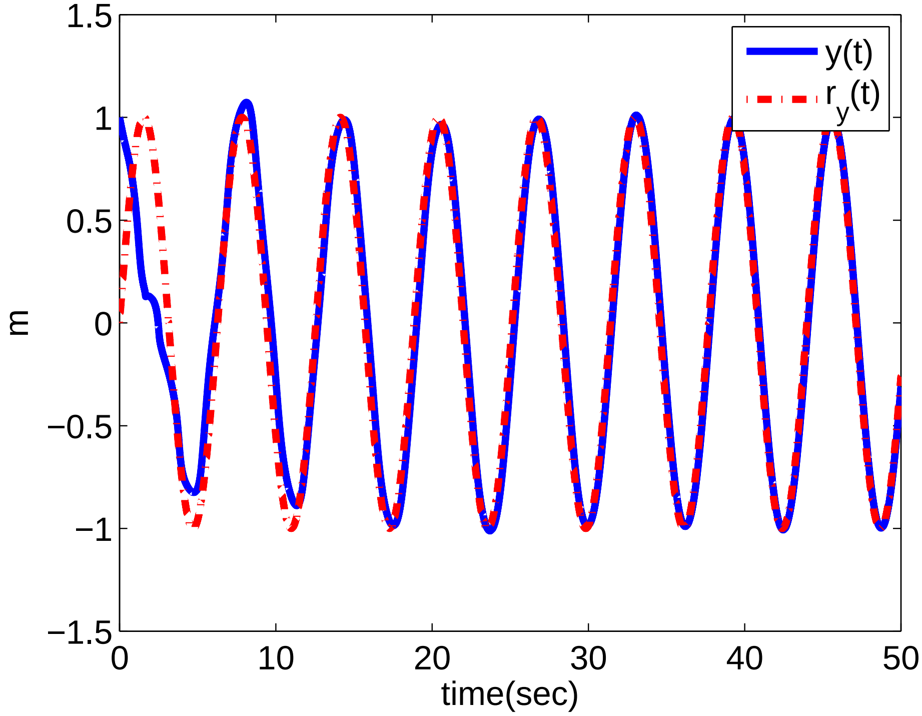

Fig. 4. $y(t)$ and $r_y(t)$

correspond to $0.1176\ddot{x}(t)+0.0235\dot{x}(t)+x(t)$, $-0.1176\ddot{x}(t)-0.0235\dot{x}(t)$, $0.0040\ddot{x}(t)-0.3424\dot{x}(t)$, $0.1176\ddot{y}(t)+0.0235\dot{y}(t)+y(t)$, $-0.1176\ddot{y}(t)-0.0235\dot{y}(t)$, $0.0040\ddot{y}(t)-0.3424\dot{y}(t)$ and $\phi(t)$, respectively. The input of this representation is $u(t)$ and the output vector is $[x(t)\ \ y(t)\ \ \phi(t)]'$ with the system matrices $A = \begin{bmatrix} A_x & \underline{0} & \underline{0} \\ \underline{0} & A_y & \underline{0} \\ \underline{0} & \underline{0} & A_\phi \end{bmatrix}$, $B = [B_x \quad B_y \quad B_\phi]'$ and $C = \begin{bmatrix} C_x & \underline{0} & \underline{0} \\ \underline{0} & C_y & \underline{0} \\ \underline{0} & \underline{0} & C_\phi \end{bmatrix}$, where $A_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.1 & -2.914 \\ 0 & 2.914 & -0.1 \end{bmatrix}$, $A_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.1 & -2.914 \\ 0 & 2.914 & -0.1 \end{bmatrix}$, $A_\phi = 0$, $B_x = [0.1176 \ -0.1176 \ 0.0040]$, $B_y = [0.1176 \ -0.1176 \ 0.0040]$, $B_\phi = 1$, $C_x = [1\ 1\ 0]$, $C_y = [-1 \ -1\ 0]$ and $C_\phi = 1$.

Hence, we need a MIMO parallel AWGN channel with 7 inputs and 7 outputs for transmitting $x_1(t)$ to $x_7(t)$, which has the following specification: $N(t)\ i.i.d. \sim N(\underline{0}, diag\{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\})$ with the power constraints: $Po_i = 1$ for $i \in \{1, 2, 3, ..., 7\}$, which meet the requirements of the propositions of Section IV.

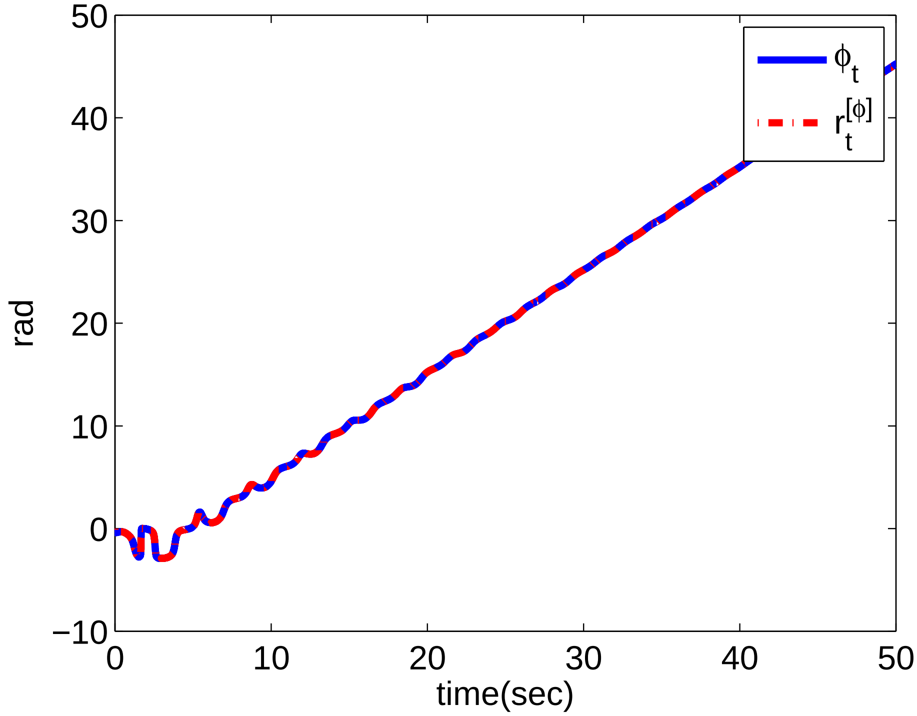

Fig. 5. $\phi(t)$ and $r_\phi(t)$

Fig. 3 to Fig. 5 illustrate that the system outputs $x(t)$, $y(t)$ and $\phi(t)$ track the reference signals $r_x(t)$, $r_y(t)$ and $r_\phi(t)$, respectively, for $\eta = 3$, $\rho = 1$, $\omega_r = 1$ and $x_r = y_r = 0$, using the technique proposed in Section V. Fig. 6 illustrates the control signal $u(t)$. Also, Fig. 7 and Fig. 8 illustrate that the autonomous vehicle tracks the reference circle. As is clear from these figures, the proposed tracking technique is able to force the nonlinear system asymptotically track the reference signals very well. The Root Cumulative Square Error (RCSE) computed for the period of [20 , 50] second (i.e., $\sqrt{\int_{20}^{50}[(x(t)-r_x(t))^2+(y(t)-r_y(t))^2]dt}$) for different chooses of ω is shown in Table I.

This table indicates that in order to have a satisfactory reference tracking, the high cut off frequency of the bandpass filter is better to be at least equals to ω_r.

To compare the performance of the proposed technique, we apply the proposed technique and the feedback linearization control technique of [35] (with the linearized system of (9) and (10) of [35]) to the block diagram of Fig. 1 for the reference signals of $r_x(t) = 0.0005t$ and $r_y(t) = 0.0002t$. The RCSE computed for the period of [20 , 50] second when the proposed technique is used is 5.43 (see Fig. 9) and the RCSE computed for the feedback linearization

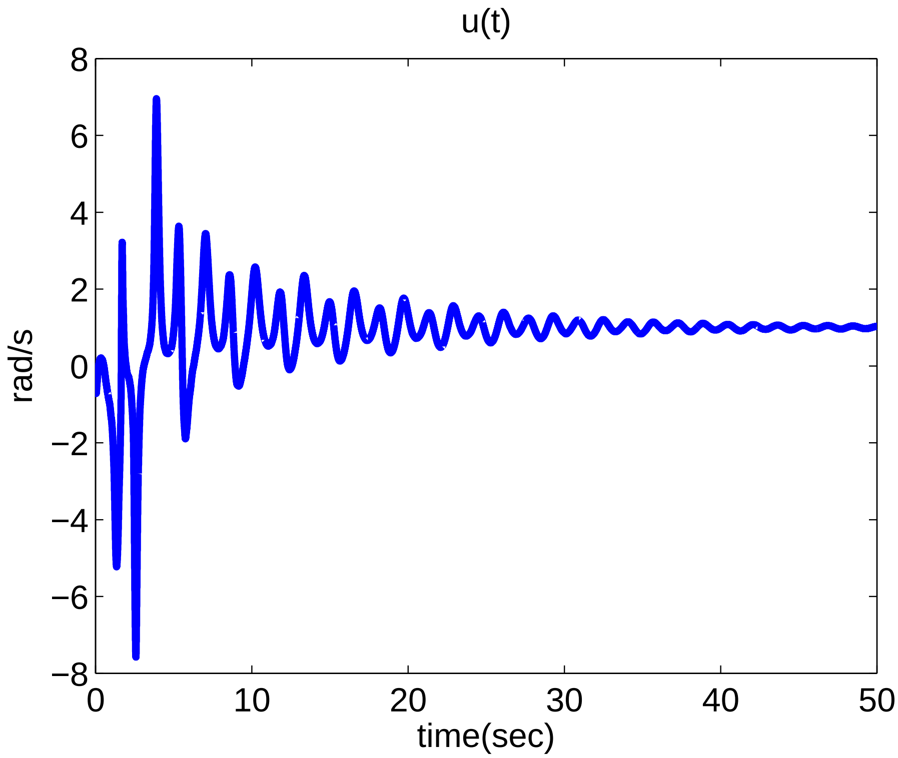

Fig. 6. Control signal: angular velocity

TABLE I

RCSE VS ω

ω	RCSE
0.2	7.91
1	5.53
10	5.59
100	5.48
1000	5.45

control technique of [35] is 17.36 (see Fig. 10). From these figures, it is clear that the proposed technique has a better performance in the presence of communication imperfections.

Over the packet erasure channel, which is subject to random packet dropout and quantization imperfections, [36] presented a novel technique, which is based on the linearization method, for the reference tracking of the unicycle model of (1). Comparing the simulation results of this section with those of [36] for tracking a circle, reveals that the approximate method presented in this paper for the unicycle model is as good as the linearization method used in [36].

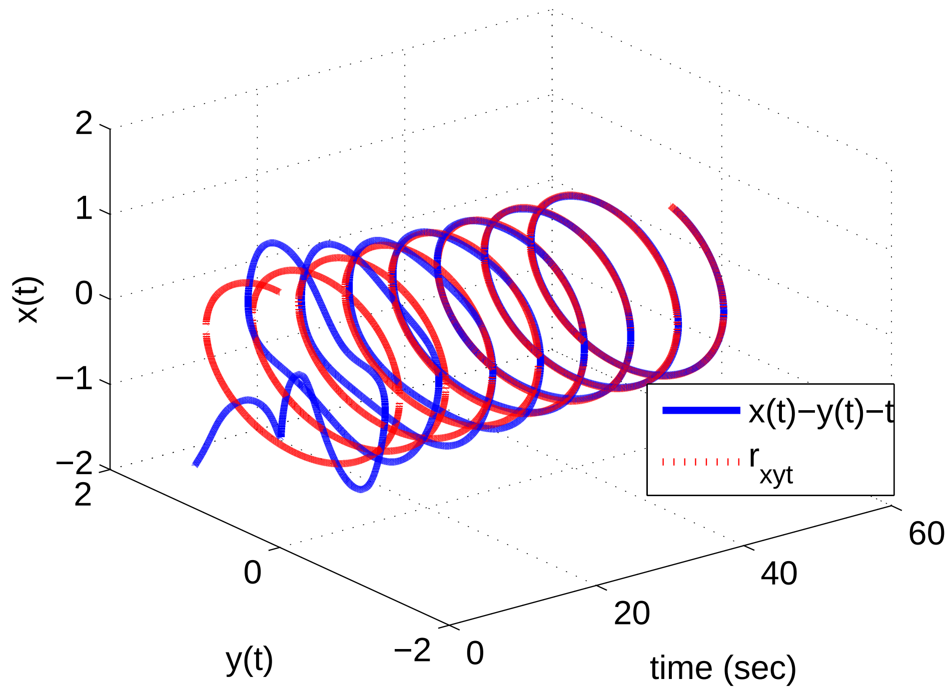

Fig. 7. $x(t) - y(t) - time$ diagram

Remark 6.1: i) Simulations study illustrates that the linear approximation method proposed in this paper which is based on the describing function is good for the particular nonlinear system considered in this paper. Although the describing function is obtained for a sinusoidal input, from Fig. 6 it is clear that for the non-sinusoidal inputs, the approximation must be also good. Otherwise, we did not get such a good reference tracking as well as state tracking for the nonlinear system (in Figs. 7, 8, 3, 4, 5) for an estimator and controller which are based on the approximate linear system.

ii) In order to utilize the communication channel and transmit with the maximum allowable powers, Po_i $(i = 1, 2)$, the encoder gain $C(t)$ is computed in Propositions 4.1 - 4.3 so that $\Big[var[C(t)(X(t) - \hat{X}(t))]\Big]_{ii} = Po_i$. However, using the proposed coding scheme and as $P(t) \to \underline{0}$, some elements of the matrix gain $C(t)$ tend to infinity. To avoid this situation, the following modifications in the encoder matrix gain $C(t)$ are suggested:

- For the encoder matrix gain of Proposition 4.1 when $p_{ii}(t) < 0.01\gamma_1\tilde{r}_i$, we replace $p_{ii}(t)$ in the matrix gain $C(t)$ by $p_{ii}(t) + 0.01\gamma_1\tilde{r}_i$.

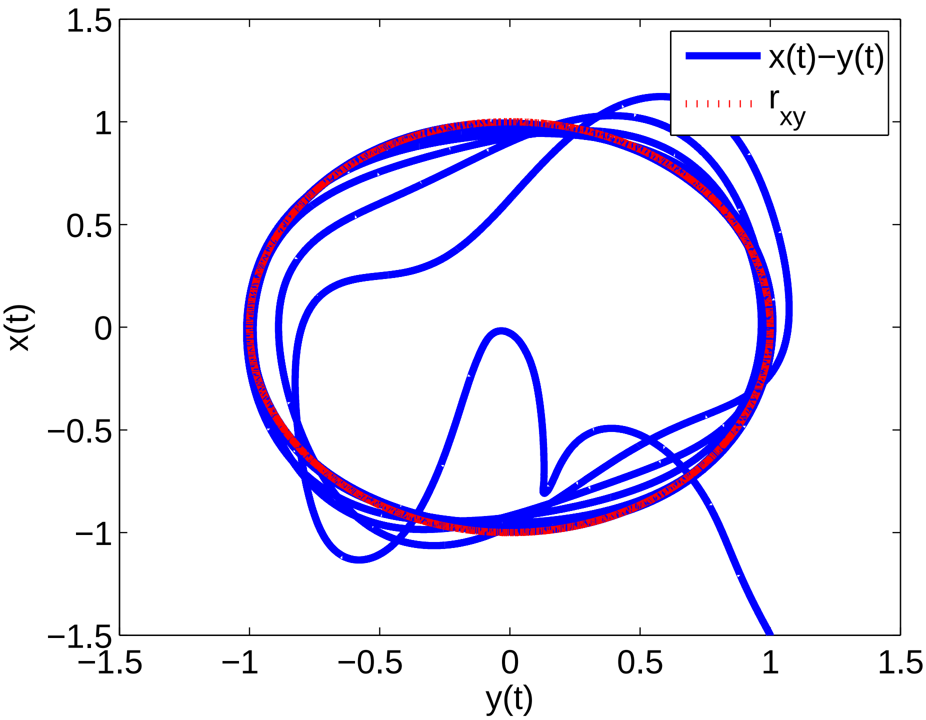

Fig. 8. $x(t) - y(t)$ diagram

- For the encoder matrix gain of Proposition 4.3, when $p_{ii}(t) < 0.01Po_i\tilde{r}_i$, we replace $p_{ii}(t)$ in the matrix gain $C(t)$ by $p_{ii}(t) + 0.01Po_i\tilde{r}_i$.
- For the encoder matrix gain of Proposition 4.2, let $h = \min(\frac{\delta\tilde{r}_1}{2}, \frac{\tilde{r}_2}{2\delta})$, then when $p_{ii}(t) < 0.01h$, we replace $p_{ii}(t)$ in the matrix gain $C(t)$ by $p_{ii}(t) + 0.01h$.

Using the above modifications, when $P(t) \rightarrow \underline{0}$, the elements of the matrix gain $C(t)$ remain bounded. We repeated computer simulation for $\omega = 1$, $\omega_r = 1$ and tracking the circle using the modified encoder matrix gain; and we observed that with the expense of transmission with the power a bit less than the maximum allowable power, we get RCSE=5.55 which is very close to the RCSE of the corresponding case with infinite gain.

VII. Conclusion and Direction for Future Research

In this paper, a new technique for mean square asymptotic state tracking and reference tracking of the autonomous vehicles over analog AWGN channel was presented. Autonomous vehicle is cascaded with a bandpass filter acting as encoder; and hence, an approximate linear dynamic system was extracted using the describing function method. Then, the results of [13] were extended

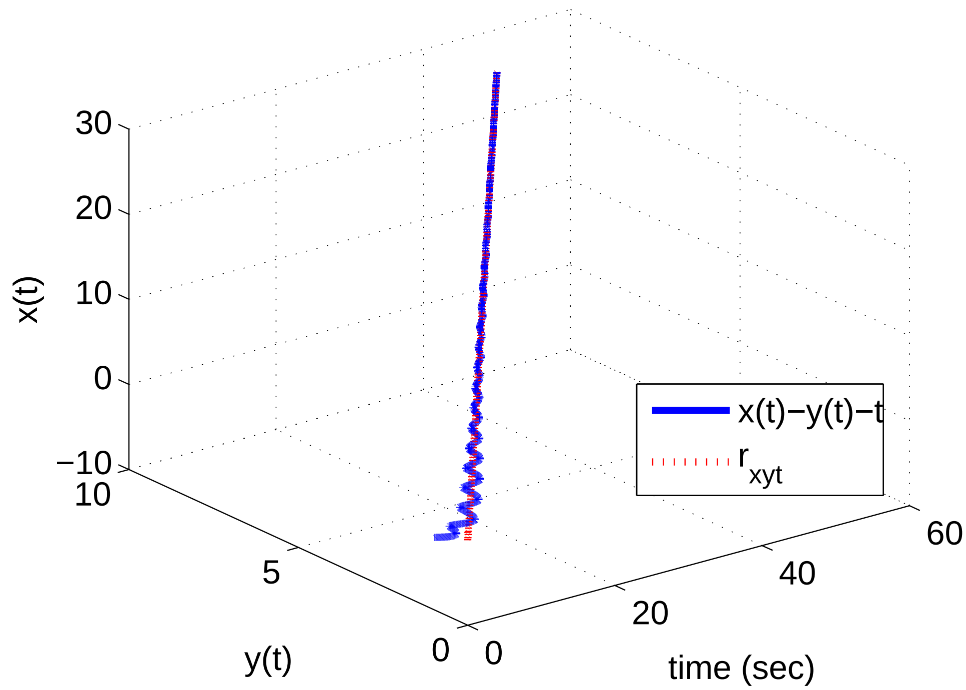

Fig. 9. $x(t) - y(t) - time$ diagram for the proposed technique

to account for systems with multiple real and non-real valued eigenvalues over MIMO parallel AWGN channel. Subsequently, by applying the extended results on the describing function, a technique for mean square asymptotic state tracking and reference tracking of autonomous vehicles was presented. Finally, the satisfactory performance of the proposed technique was illustrated using computer simulations.

In addition to the describing function method, there are other methods, e.g., based on the Volterra series theorem [37], [38], [39] for nonlinear analysis in the frequency domain. Using the Volterra series expansion, the Generalized Frequency Response Function (GFRF) was defined in [40], which is multivariate Fourier transform of the Volterra kernels. This provides a useful concept for approximation of the nonlinear systems. Therefore, for future, it is interesting to compare the performance of the proposed technique with those based on other approximation methods (e.g., GFRF). In addition, it is interesting to consider more realistic model for the dynamic system (1), e.g., a dynamic system subject to measurement and process noises. This research direction is currently under investigation in our research team.

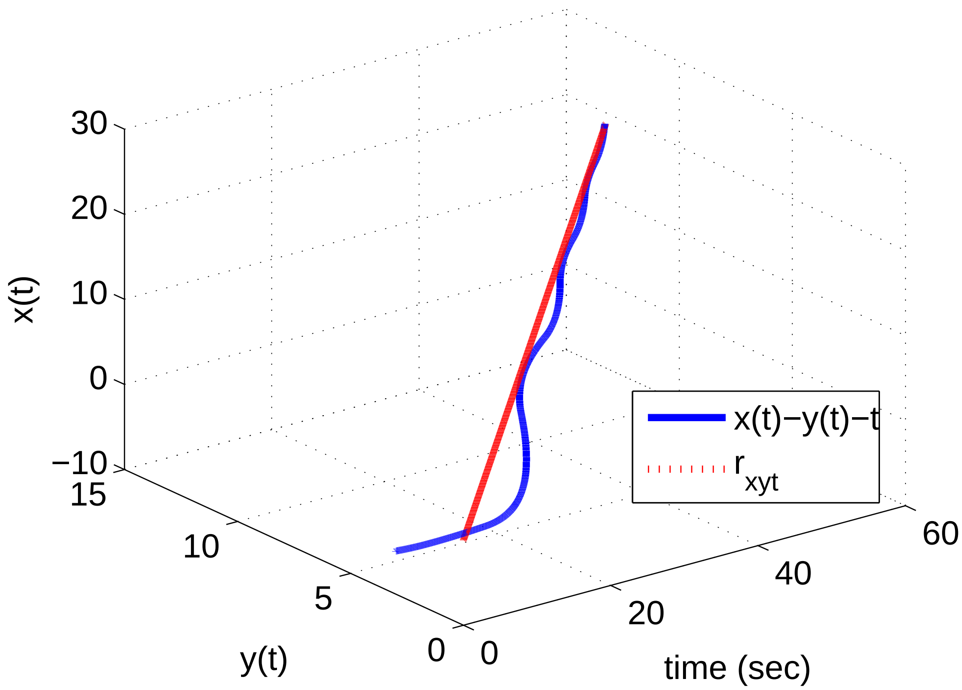

Fig. 10. $x(t) - y(t) - time$ diagram for the feedback linearization technique of [35]

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Biography

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