Telemetry and Tele-operation of Nonlinear Systems: Applications in Tele-operation of Autonomous Vehicles

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***Abstract*—This paper presents a new technique for state and reference tracking of nonlinear dynamics systems over the packet erasure channel, which is an abstract model for transmission via the Zigbee modules. A proper encoder, decoder and controller for state and reference tracking of nonlinear dynamics systems when measurements are sent through the limited capacity erasure channel, are presented. Then, the satisfactory performance of the proposed technique is illustrated via computer simulations by applying this technique on the unicycle model, which represents the dynamics of autonomous vehicles.**

***Index Terms*—Networked control system, nonlinear systems, the packet erasure channel, the unicycle model**

I. Introduction

Research on real time state estimation at the end of communication links (known also as state tracking or telemetry) and stability over communication channels subject to limited capacity constraint is concerned with situations involving dynamics systems controlled over limited capacity communication links which can be also corrupted by noise. Fig. 1 illustrates a basic block diagram considered in the literature for studying the problem of state tracking and stability subject to limited capacity constraint. In this block diagram there is a limited capacity communication link from sensors to remote controller; while the connection from controller to the system is direct. The limitation on transmission capacity results in distortion on the measurements that must be compensated by designing proper encoder and decoder for real time reliable data reconstruction of measurements at the end of communication link.

In the literature (e.g, [1]–[11]) the authors considered state tracking and/or stability problems of linear systems over communication channels subject to imperfections (e.g., limited capacity, noise, etc.), whereas most of important applications of networked control systems, such as tele-operation of autonomous vehicles, involve nonlinear systems. The works on state tracking and control of nonlinear systems over communication channels subject to imperfections are limited to state tracking and/or stability for the digital noiseless channel ([12], [13] and [14]), AWGN channel ([15]) or the nonlinear Lipschitz dynamics systems ([14] and [16]). Nevertheless, in some emerging applications, such as tele-operation of miniature autonomous vehicles (e.g., drones), we deal with nonlinear dynamics systems which are more complicated [17] to be modeled by the Lipschitz systems. Also, in this applications, the communication from the system to remote base station, where the remote controller is located, is mostly via low power Zigbee modules; and there is limitation on the length of transmitted packet for each measurement in order to save the on board battery power as much as possible. Hence, in these applications, the communication link must be modeled by the limited capacity erasure channel, i.e., by the packet erasure channel with feedback acknowledgment. Motivated by these applications; this paper addresses the problem of state and reference tracking (and hence stability) of nonlinear systems over the packet erasure channel with feedback acknowledgment, as is shown in the block diagram of Fig. 1. In the aforementioned application, the communication from remote base station to the dynamics systems can be performed with high transmission power; and hence, in the block diagram of Fig. 1, the communication link from remote controller to the system can be considered without imperfections.

For the block diagram of Fig. 1, which can correspond to the tele-operation system of autonomous vehicles over Zigbee, a proper encoder, decoder and controller for state and reference tracking of nonlinear dynamics systems by remote controller are presented when measurements are sent through the packet erasure channel. Then, the satisfactory performance of the proposed technique is illustrated via computer simulations by applying this technique on the unicycle model, which represents the dynamics of autonomous vehicles.

The paper is organized as follows. In Section II, the problem formulation is presented. Section III is devoted

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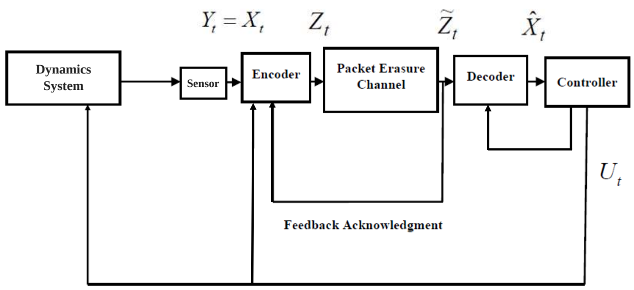

Fig. 1. A dynamics system controlled over the packet erasure channel with feedback acknowledgment.

to the design of a proper encoder, decoder and controller for state and reference tracking of nonlinear dynamics systems over the packet erasure channel. Section IV is devoted to the simulation results for the unicycle model. Finally, the paper is concluded by summarizing the contributions of the paper and direction for future research in Section V.

II. Problem Formulation

Throughout, certain conventions are used: $|\cdot|$ denotes the absolute value, $\|\cdot\|$ the Euclidean norm and V' denotes the transpose of vector/matrix V. A^{-1} denotes the inverse of a square matrix A. '$\doteq$' means 'by definition is equivalent to' and $Z^t \doteq (Z_1, Z_2, ..., Z_t)$. $\mathbb{R}$ denotes the set of real numbers and $N_+ \doteq \{0, 1, 2, 3, ...\}$. Also, $\underline{0}$ denotes the zero vector/matrix.

The building blocks of Fig. 1 are described below:

Dynamics System: The dynamics system is described by the following nonlinear discrete time system:

$$\begin{cases} X_{t+1} = F(X_t) + BU_t \\ Y_t = X_t \end{cases} \quad (1)$$

where $t \in N_+$ is the time instant, $F(X_t) = \begin{bmatrix} f_1(X_t) & f_2(X_t) & \dots & f_n(X_t) \end{bmatrix}' \in \mathbb{R}^n$ is a nonlinear continuous function, $X_t = \begin{bmatrix} x_t^{(1)} & x_t^{(2)} & \dots & x_t^{(n)} \end{bmatrix}' \in \mathbb{R}^n$ is the vector of states of the system, $Y_t \in \mathbb{R}^n$ is the observation signal, $U_t \in \mathbb{R}^m$ is the control signal. Throughout, it is assumed that the probability measure associated with the initial state X_0 with components $x_0^{(i)}$, $i = \{1, 2, ..., n\}$, has bounded support. That is, for each $i \in \{1, 2, ..., n\}$ there exists a compact set $[-L_0^{(i)}, L_0^{(i)}] \in \mathbb{R}$ such that $\Pr(x_0^{(i)} \in [-L_0^{(i)}, L_0^{(i)}]) = 1$. Note that X_0 is unknown for decoder and controller.

Communication Channel: Communication channel between the system and controller is a limited capacity erasure channel with feedback acknowledgment. It is a digital channel that transmits a packet of binary data with the limited length at each channel use. The channel input and output alphabets are denoted by $\mathcal{Z}$ and $\tilde{\mathcal{Z}}$, respectively; and Z_t denotes the channel input at time instant $t \in N_+$, which is a packet of binary data with length R containing information bits. Also $\tilde{Z}_t$ denotes the corresponding channel output. Let e denote the erasure symbol. Then,

$$\tilde{Z}_t = \begin{cases} Z_t & with\ probability\ 1-\alpha \\ e & with\ probability\ \alpha \end{cases} \quad (2)$$

That is, this channel erases a transmitted packet with probability α. Throughout, it is assumed that the erasure probability α is known a priori. In the channel considered in this paper, there are feedback acknowledgments from receiver to encoder. That is, if a transmission is successful, an acknowledgment bit is sent from receiver to encoder indicating that the transmission was successful. The packet erasure channel with feedback acknowledgment is an abstract model for the commonly used data transmission technologies, such as the Internet, WiFi and Zigbee. The capacity of this channel is $(1-\alpha)R$ bits/time step.

To compensate the imperfections on the received measurements which are due to random packet dropout and distortion caused by the limitation on channel capacity, we need to use a proper encoder and decoder. Encoder and decoder considered in this paper have the following general description.

Encoder: Encoder is a causal operator denoted by $Z_t = \mathcal{E}(Y_t, \tilde{Z}^{t-1}, U^{t-1})$ that maps the system output Y_t (by the knowledge of the past channel outputs and control signals) to the channel input Z_t, which is a string of binaries with length R.

Decoder: Decoder is a causal operator denoted by $\hat{X}_t = D(\tilde{Z}^t, U^{t-1})$ that maps the channel output to $\hat{X}_t$, which is the estimate of the states variable at the decoder.

Controller: Controller has the following structure $U_t = -B'(BB')^{-1}(F(\hat{X}_t) - \mathcal{R}_{t+1})$, where $\mathcal{R}_{t+1}$ is the reference signal. Note that for the stability purposes, we set $\mathcal{R}_t = \underline{0}$.

The objective of this paper is to design an encoder, decoder and a controller that result in almost sure asymptotic state and reference tracking (and hence stability) of the system (1), as defined below:

Definition 2.1: (Almost Sure Asymptotic State Tracking): Consider the block diagram of Fig. 1 described by the nonlinear dynamics system (1) over the packet erasure channel, as described above. It is said that the states are almost sure asymptotically tracked if there exist an encoder and a decoder such that the following property holds: $\Pr(\lim_{t\to\infty} \|X_t - \hat{X}_t\| = 0) = 1$.

Definition 2.2: (Almost Sure Asymptotic Stability): Consider the block diagram of Fig. 1 described by the nonlinear dynamics system (1) over the packet erasure channel, as described above. It is said that the system is almost sure asymptotically stable if there exist an encoder, decoder and a controller such that the following property holds: $\Pr(\lim_{t\to\infty} \|X_t\| = 0) = 1$.

Definition 2.3: (Almost Sure Asymptotic Reference Tracking): Consider the block diagram of Fig. 1 described by the nonlinear dynamics system (1) over the packet erasure channel, as described above. It is said that the system is almost sure asymptotically track the reference signal $\mathcal{R}_t \in \mathbb{R}^n$ if there exist an encoder, decoder and a controller such that the following property holds: $\Pr(\lim_{t\to\infty} \|X_t - \mathcal{R}_t\| = 0) = 1$.

Remark 2.4: Note that the stability is a special case of the reference tracking with $\mathcal{R}_t = \underline{0}$.

III. Encoder, Decoder and Controller

In this section, we are concerned with the dynamics system (1). We first present an encoder, decoder and a sufficient condition on the length of transmitted packets R, under which the states of the system almost sure asymptotically are estimated at the end of communication link. To achieve this goal and for the simplicity of presentation, we first suppose that $F(X)$ in (1) is monotone and scalar function. The extensions of this result to more general case of non-monotone and vector function $F(X)$ are straight forward. Subsequently, we show that using the proposed structure for the controller, the reference tracking and stability are also achieved.

Now, suppose the nonlinear function $F(X)$ is strictly monotone and Scalar. Also, suppose X_t, $U_t \in \mathbb{R}$ $(n, m = 1)$. The proposed coding scheme for the scalar case works as follows:

We fix the rate to be R. At the time instant $t = 0$, we notice that $X_0 \in [-L_0, L_0]$. The encoder and decoder partition the interval $[F(-L_0), F(L_0)]$ into 2^R equal sized, non-overlapping sub-intervals and the center of each sub-interval is denoted by $\eta_0, \eta_1, ..., \eta_{2^R-1}$ (see Fig. 2). Then, the projection of η_is in the X-axes is computed and denoted by $\gamma_0 = F^{-1}(\eta_0), \gamma_1 = F^{-1}(\eta_1), ..., \gamma_{2^R-1} = F^{-1}(\eta_{2^R-1})$, where $F^{-1}(.)$ is the inverse function of $F(.)$. Subsequently, the index of the sub-interval that includes X_0 (e.g., γ_{j0} where $j0 \in \{0, 1, ..., 2^R - 1\}$) is encoded into R bits and transmitted to the decoder through the packet erasure channel (see Fig. 2). If the decoder receives this R bits successfully, it identifies the index of the sub-interval where X_0 lives in; and the value of this index is chosen as $\hat{X}_0$ (e.g., γ_{j0} where $j0 \in \{0, 1, ..., 2^R - 1\}$). But, if erasure occurs, then $\hat{X}_0 = 0$. Hence, for this time instant, the decoding error is bounded above by

$$|X_0 - \hat{X}_0| \leq V_0 = M_0 L_0;$$
$$M_0 = \begin{cases} \frac{1}{2^R}, & \Pr(M_0 = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_0 = 1) = \alpha \end{cases} \tag{3}$$

At the time instant $t = 1$, using feedback acknowledgment, the encoder can compute $\hat{X}_0$; and therefore, it encodes $X_1 - F(\hat{X}_0) - BU_0$. To encode this signal, the interval $[-L_1, L_1]$ is computed by the encoder and decoder as follows:

$$\begin{aligned} |X_1 - \dot{\hat{X}}_1| &= |X_1 - F(\hat{X}_0) - BU_0| \\ &= |F(X_0) + BU_0 - F(\hat{X}_0) - BU_0| \\ &= |F(X_0) - F(\hat{X}_0)| \\ &\leq M_0 \frac{|F(L_0) - F(-L_0)|}{2} \doteq L_1 \end{aligned} \tag{4}$$

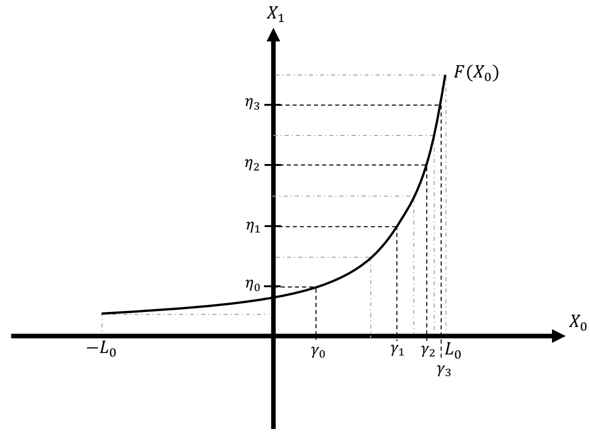

Fig. 2. Equal sized, non-overlapping sub-intervals for encoding with $R = 2$.

Then, similar to the previous case, encoder and decoder partition the interval $[F(-L_1 + F(\hat{X}_0) + BU_0), F(L_1 + F(\hat{X}_0) + BU_0)]$ into 2^R equal sized, non-overlapping sub-intervals and the inverse of the center of each sub-interval is chosen as the index of that interval. When the encoder observes the signal $X_1 - F(\hat{X}_0) - BU_0$, the index of the sub-interval that includes $X_1 - F(\hat{X}_0) - BU_0$ (e.g., γ_{j1} where $j1 \in \{0, 1, ..., 2^R - 1\}$) is encoded into R bits and transmitted to the decoder through the packet erasure channel. Then, the decoder constructs $\hat{X}_1$ as follows:

$$\hat{X}_1 = \begin{cases} \gamma_{j1}, & \Pr(M_1 = \frac{1}{2^R}) = 1 - \alpha \\ F(\hat{X}_0) + BU_0, & \Pr(M_1 = 1) = \alpha \end{cases} \tag{5}$$

Therefore, for this case, the decoding error is bounded above by

$$|X_1 - \hat{X}_1| \leq V_1 = M_1 L_1;$$
$$M_1 = \begin{cases} \frac{1}{2^R}, & \Pr(M_1 = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_1 = 1) = \alpha \end{cases} \tag{6}$$

Similarly, for the rest of time instants $t > 1$, the encoder encodes $X_t - F(\hat{X}_{t-1}) - BU_{t-1}$. To encode this signal the interval $[-L_t, L_t]$ is chosen as follows:

$$\begin{aligned} &|X_t - \dot{\hat{X}}_t| = |X_t - F(\hat{X}_{t-1}) - BU_{t-1})| \\ &= |F(X_{t-1}) + BU_{t-1} - F(\hat{X}_{t-1}) - BU_{t-1}| \\ &= |F(X_{t-1}) - F(\hat{X}_{t-1})| \\ &\leq M_{t-1} \frac{|F(L_{t-1} + \Delta_{t-2}) - F(-L_{t-1} + \Delta_{t-2})|}{2} \doteq L_t; \end{aligned}$$
$$M_{t-1} = \begin{cases} \frac{1}{2^R}, & \Pr(M_{t-1} = \frac{1}{2^R}) = 1 - \alpha \\ 1, & \Pr(M_{t-1} = 1) = \alpha \end{cases} \tag{7}$$

where $\Delta_{t-2} \doteq F(\hat{X}_{t-2}) + BU_{t-2}$.

Then, the encoder and decoder partition the interval $[F(-L_t + F(\hat{X}_{t-1}) + BU_{t-1}), F(L_t + F(\hat{X}_{t-1}) + BU_{t-1})]$ into 2^R equal sized, non-overlapping sub-intervals and the inverse of the center of each sub-interval is chosen as the index of that interval. When the

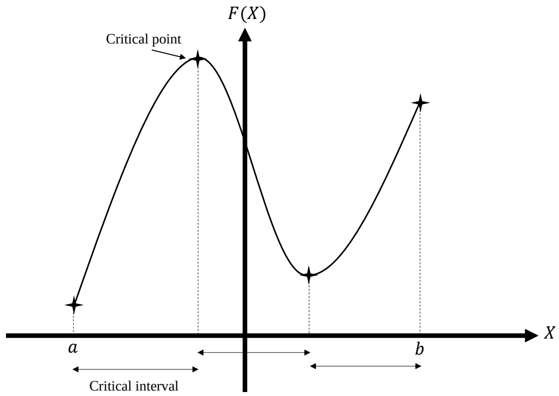

Fig. 3. Critical points and critical intervals.

encoder observes the signal $X_t - F(\hat{X}_{t-1}) - BU_{t-1}$, the index of the sub-interval that includes $X_t - F(\hat{X}_{t-1}) - BU_{t-1}$ (e.g., γ_{jt}) is encoded into R bits and transmitted to the decoder through the packet erasure channel. Then the decoder constructs $\hat{X}_t$ as below:

$$\hat{X}_t = \begin{cases} \gamma_{jt}, & \Pr(M_{t-1} = \frac{1}{2^R}) = 1 - \alpha \\ F(\hat{X}_{t-1}) + BU_{t-1}, & \Pr(M_{t-1} = 1) = \alpha \end{cases} \tag{8}$$

By following a similar procedure, as described above, the sequence $\hat{X}_0, \hat{X}_1, \hat{X}_2, ...$ are constructed at the decoder.

Let $\Gamma_{Max} \doteq \max_{X,Y \in [-L_0, L_0]} \frac{|F(X) - F(Y)|}{|X - Y|}$ and $\Gamma_t \doteq \frac{|F(L_t + F(\hat{X}_{t-1}) + BU_{t-1}) - F(-L_t + F(\hat{X}_{t-1}) + BU_{t-1})|}{2L_t}$. Now, we must show that the above coding scheme results in almost sure asymptotic state tracking. This result is shown in the following proposition.

Proposition 3.1: Consider the control system of Fig. 1 described by the dynamics system (1) over the packet erasure channel with erasure probability α and feedback acknowledgment, as described earlier. Suppose that the transmission rate R satisfies the following inequality:

$$(1 - \alpha)R > \max\{0, \log_2 \Gamma_{Max}\} \tag{9}$$

Then, using the proposed encoding and decoding scheme, we have almost sure asymptotic state tracking in the form of $\hat{X}_t \rightarrow X_t$, P-a.s.; or equivalently, $\Pr(\lim_{t\to\infty} \|X_t - \hat{X}_t\| = 0) = 1$.

Proof: : It follows from the strong law of large numbers [18]. Due to the page limitation, the detailed proof is omitted.

For the non-monotone case, let us refer to the collection of the extremum points and $F(a)$ and $F(b)$, where $X \in [a, b]$ as the critical points of the function $F(X)$ (see Fig. 3). Now, we have the following corollary that extends the previous result to non-monotone function $F(X)$:

Corollary 3.2: For the non-monotone function $F(X)$, we consider the intervals between each critical point and call them critical intervals. Then, by applying the similar procedure as described above to each critical interval and by adding some extra bits to the transmitted packet for identification of the critical interval where X_t is located on, it can be shown that the almost sure asymptotic state tracking is achieved.

Remark 3.3: By following a similar procedure, the above results are extended to the vector function $F(X)$.

Now in the following Proposition, we show that the proposed coding scheme combined by the controller $U_t = -B'(BB')^{-1}(F(\hat{X}_t) - \mathcal{R}_{t+1})$ result in the reference tracking.

Proposition 3.4: Controller with the following structure $U_t = -B'(BB')^{-1}(F(\hat{X}_t) - \mathcal{R}_{t+1})$ results in almost sure reference tracking of the system (1).

Proof: From (1) we have $X_{t+1} = F(X_t) - (BB')(BB')^{-1}(F(\hat{X}_t) - \mathcal{R}_{t+1}) = F(X_t) - F(\hat{X}_t) + \mathcal{R}_{t+1}$. Now, as $\hat{X}_t \rightarrow X_t$ for $t \rightarrow \infty$, P-a.s., $F(X_t) - F(\hat{X}_t) + \mathcal{R}_{t+1} \rightarrow \mathcal{R}_{t+1}$, P-a.s.; and therefore, reference tracking (and hence stability for $\mathcal{R}_t = 0$) is achieved. This completes the proof.

IV. Simulation Results

In this section, for the purpose of illustration, we apply the proposed encoder, decoder and controller to the nonlinear dynamics of miniature drones, autonomous road vehicles and autonomous under water vehicles that can be modeled by the unicycle model [17]. The dynamics of each miniature drones, autonomous road vehicles and autonomous under water vehicles are described by a 6 degrees of freedom model. However, the vehicles dynamics can be handled by local control loops, which results in a kinematic unicycle model, as follows [17]:

$$\begin{cases} \dot{x}(t) = v(t)\cos(\phi(t)) \\ \dot{y}(t) = v(t)\sin(\phi(t)) \\ \dot{\phi}(t) = u(t) \end{cases} \tag{10}$$

where $x(t)$, $y(t)$ are the position vector, $\phi(t)$ the heading angle, and the control inputs are the vehicle forward velocity $v(t)$ and the turning rate $u(t)$. The state vector of the system is $X(t) = \begin{bmatrix} x(t) & y(t) & \phi(t) \end{bmatrix}'$ and the input vector is $U(t) = \begin{bmatrix} v(t) & u(t) \end{bmatrix}'$. The discrete time equivalent model is described by (11), where T is the sampling period.

$$\begin{cases} x_{t+1} = x_t + Tv_t\cos(\phi_t) \\ y_{t+1} = y_t + Tv_t\sin(\phi_t) \\ \phi_{t+1} = \phi_t + Tu_t \end{cases} \tag{11}$$

In this model x_t, y_t, ϕ_t, v_t and u_t are the discrete time equivalent signals of $x(t)$, $y(t)$, $\phi(t)$, $v(t)$ and $u(t)$, respectively. Note that for this model, the state vector is $X_t = \begin{bmatrix} x_t & y_t & \phi_t \end{bmatrix}' \doteq \begin{bmatrix} x_t^{(1)} & x_t^{(2)} & x_t^{(3)} \end{bmatrix}'$.

Therefore, the state space representation of the equivalent discrete time system has the following form:

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \phi_{t+1} \end{bmatrix} = \begin{bmatrix} x_t + Tv_t\cos(\phi_t) \\ y_t + Tv_t\sin(\phi_t) \\ \phi_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} u_t \tag{12}$$

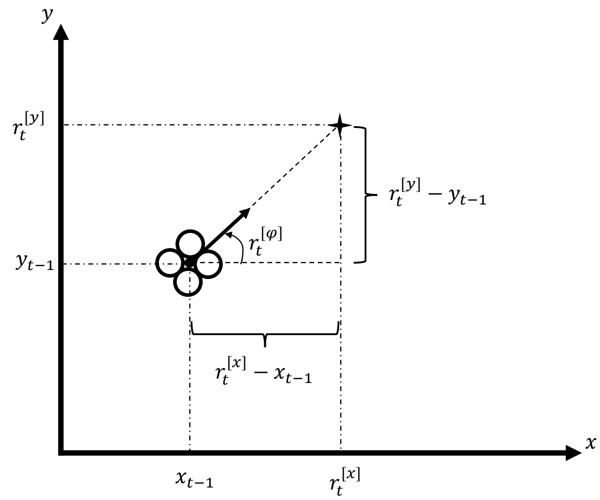

Fig. 4. An autonomous vehicle with positions x_{t-1} and y_{t-1} moving towards the desired positions $r_t^{[x]}$ and $r_t^{[y]}$.

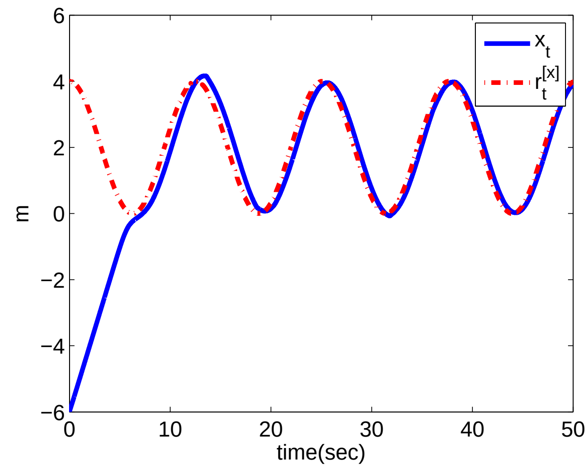

Fig. 5. x_t and $r_t^{[x]}$ for $\alpha = 0.9$.

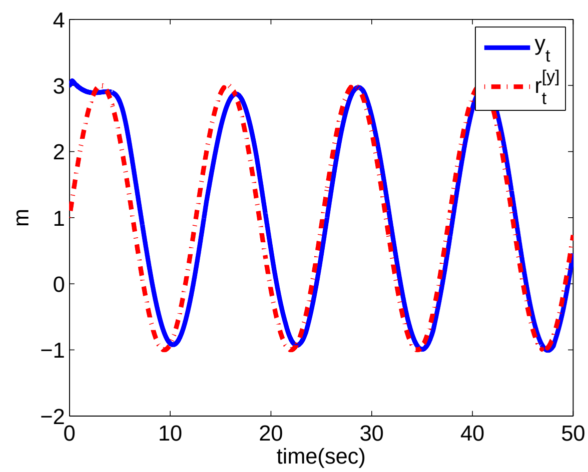

Fig. 6. y_t and $r_t^{[y]}$ for $\alpha = 0.9$.

which is in the form of the system (1) with $F(X_t) = \begin{bmatrix} x_t + Tv_t\cos(\phi_t) & y_t + Tv_t\sin(\phi_t) & \phi_t \end{bmatrix}' \doteq \begin{bmatrix} f_1(X_t) & f_2(X_t) & f_3(X_t) \end{bmatrix}'$ and $B = \begin{bmatrix} 0 & 0 & T \end{bmatrix}'$. The autonomous vehicle must track a circle with the center located at (x_r, y_r) and the radius of ρ with the angular velocity of ω_r. Therefore, $\begin{bmatrix} x_t & y_t & \phi_t \end{bmatrix}'$ must track the reference signal $\begin{bmatrix} r_t^{[x]} & r_t^{[y]} & r_t^{[\phi]} \end{bmatrix}'$, where $r_x(t) = x_r + \rho\cos(\omega_r Tt)$, $r_t^{[y]} = y_r + \rho\sin(\omega_r Tt)$ and $r_t^{[\phi]} = \arctan(\frac{r_t^{[y]} - \hat{y}_{t-1}}{r_t^{[x]} - \hat{x}_{t-1}})$ (see Fig. 4). Note that for the simplicity of design, we choose the forward velocity constant and equals to $v(t) = 1\ m/s$. Therefore, for tracking a circle with the center located at $(2, 1)$ and the radius of 2, by the autonomous vehicle, we choose $\mathcal{R}_t \doteq \begin{bmatrix} r_t^{[x]} & r_t^{[y]} & r_t^{[\phi]} \end{bmatrix}' = \begin{bmatrix} 2 + 2\cos(0.5Tt) & 1 + 2\sin(0.5Tt) & \arctan(\frac{r_t^{[y]} - \hat{y}_{t-1}}{r_t^{[x]} - \hat{x}_{t-1}}) \end{bmatrix}'$ as the reference signals. For simulations, we also choose $T = 0.01\,\text{sec}$, $x_0, y_0 \in [-10, 10]$, $\phi_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\alpha = 0.9$, which indicates that 90 percent of the transmitted packets are dropped. Also, for designing the controller, we use the pseudo inverse of BB' by computing its singular values. Fig. 5 to Fig. 10 illustrate the results of the simulations. They illustrate that the desired tracking is achieved although 90 percent of the transmitted packets are dropped.

To compare the performance of the proposed technique, we apply the proposed technique and the feedback linearization control technique of [19] (with the linearized system of (9) and (10) of [19]) to the block diagram of Fig. 1 with the unicycle model of (11) as the dynamics system with the reference signals of $r_t^{[x]} = 0.05Tt$ and $r_t^{[y]} = 0.02Tt$ ($T = 0.01\,\text{sec}$) and the following initial conditions: $x_0, y_0 \in [-10, 10]$ and $\phi_0 \in [-2, 2]$. The Root Sum Square Error (RSSE) computed from the sample $t = 30/T$ to the sample $t = 100/T$ ($30\,\text{sec}$. to $100\,\text{sec}$.) for $\alpha = 0.5$, $\alpha = 0.9$ and $\alpha = 0.98$ when the proposed technique is used is shown in the following Table.

α	RSSE
0.5	1.93
0.9	2.86
0.98	6.44

The following table also shows the RSSE computed for the feedback linearization control technique of [19].

α	RSSE
0.5	30.23
0.9	230.75
0.98	617.14

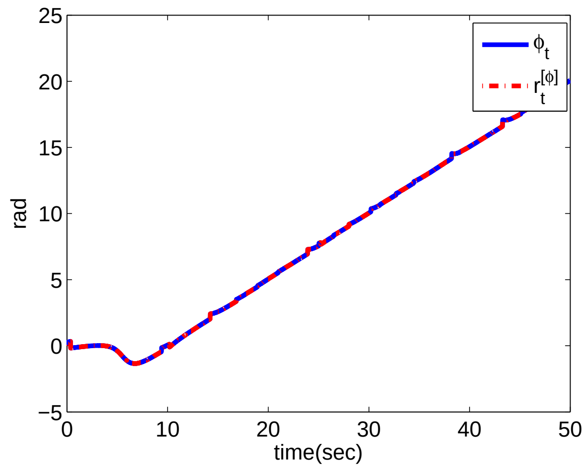

Fig. 7. ϕ_t and $r_t^{[\phi]}$ for $\alpha = 0.9$.

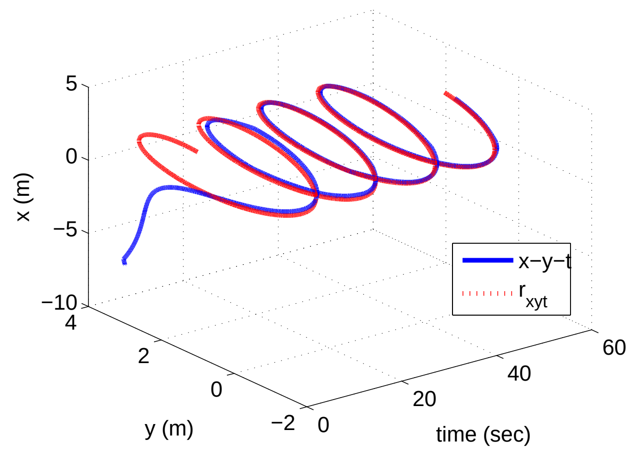

Fig. 9. $x_t - y_t - time$ diagram for $\alpha = 0.9$.

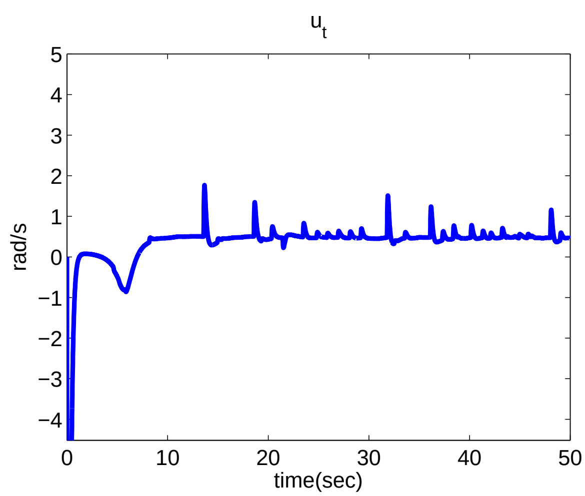

Fig. 8. Control signal: angular velocity.

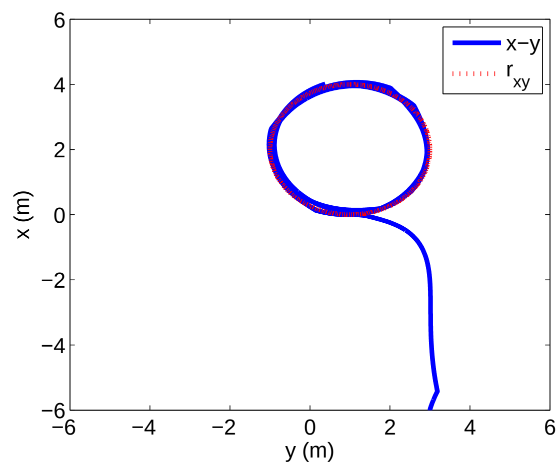

Fig. 10. $x_t - y_t$ diagram for $\alpha = 0.9$.

For $\alpha = 0.9$ the performances of the proposed technique and the feedback linearization technique of [19] are also illustrated in Fig. 11 and Fig. 12, respectively. From these tables and figures, it is clear that the proposed technique has a better performance.

V. Conclusion and Direction for Future Research

This paper presented a new technique for state and reference tracking of nonlinear systems by remote controller over the packet erasure channel. A proper encoder, decoder and controller for state and reference tracking of nonlinear systems when measurements are sent through the limited capacity erasure channel were presented. Then, the satisfactory performance of the proposed state and reference tracking technique was illustrated via computer simulations by applying this technique on the unicycle model, which represents the dynamics of autonomous vehicles.

The proposed scheme can be applied to nonlinear systems with bounded Γ_{Max}. For future, it is interesting to relax this assumption.

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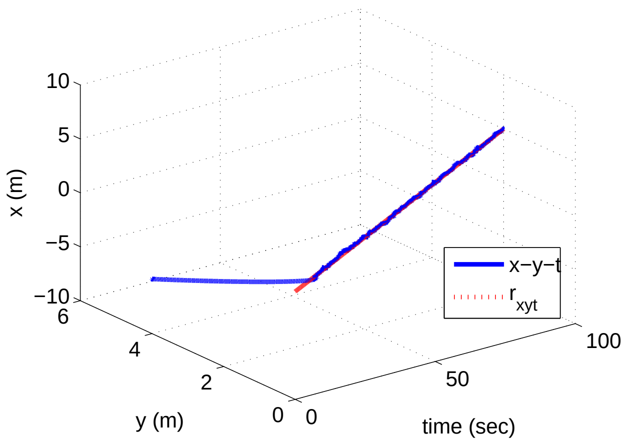

Fig. 11. $x_t - y_t - time$ diagram for $\alpha = 0.9$ when the proposed technique is applied.

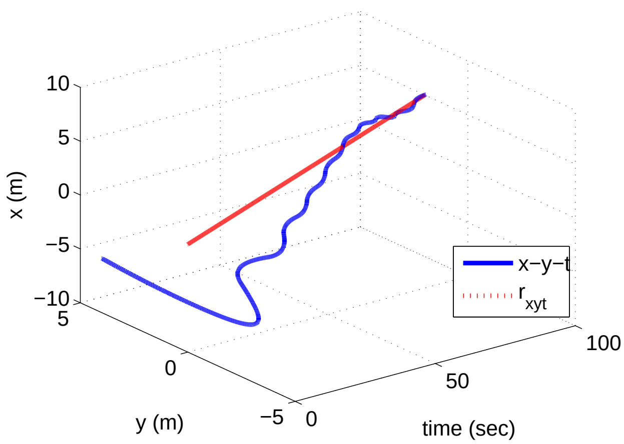

Fig. 12. $x_t - y_t - time$ diagram for $\alpha = 0.9$ when the feedback linearization technique of [19] is applied.

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