Reference Tracking of Nonlinear Dynamic Systems over AWGN Channel Using Describing Function

Ali Parsa and Alireza Farhadi

Abstract

This paper presents a new technique for mean square asymptotic reference tracking of nonlinear dynamic systems over Additive White Gaussian Noise (AWGN) channel. The nonlinear dynamic system has periodic outputs to sinusoidal inputs and is cascaded with a bandpass filter acting as encoder. Using the describing function method, the nonlinear dynamic system is represented by an equivalent linear dynamic system. Then, for this system, a mean square asymptotic reference tracking technique including an encoder, decoder and a controller is presented. It is shown that the proposed reference tracking technique results in mean square asymptotic reference tracking of nonlinear dynamic systems over AWGN channel. The satisfactory performance of the proposed reference tracking technique is illustrated using practical example simulations.

Keywords- Networked control system, nonlinear dynamic system, the describing function.

I. Introduction

A. Motivation and Background

One of the issues that has begun to emerge in a number of applications, such as tele-operation of micro autonomous vehicles [1] and smart oil drilling system [2] is how to stabilize a dynamic system over communication channels subject to imperfections (e.g., noise, packet dropout, distortion). This motivates research on stability problem of dynamic systems over communication channels subject to imperfections. Some results addressing basic problems in stability of dynamic systems over communication channels subject to imperfections can be found in [3]-[20]. In most of these references, the goal is the reliable data reconstruction and/or stability over (noisy) communication channels subject to limited capacity constraint. Intuitively, the results

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in this direction provide a quantitative understanding of the way in which restriction on the data rate of the exchanged information among components of the system, degrades the performance of the system. In other direction (e.g., [21]), the goal is to stabilize a dynamical system over communication channels subject to noise, delay or lost, while there is no restriction on the transmission data rate.

Dynamic systems can be viewed as continuous alphabet information sources with memory. Therefore, many works in the literature (e.g., [3], [9], [10], [16]-[20]) are dedicated to the question of optimality and stability over Additive White Gaussian Noise (AWGN) channel, which itself is naturally a continuous alphabet channel [22]. [9], [10] addressed the problem of mean square stability and tracking of linear Gaussian dynamic systems over AWGN channel when noiseless feedback channel is available full time and they assumed that the communication of control signal from remote controller to system is perfect. In [9], the authors presented an optimal control technique for asymptotic bounded mean square stability of a partially observed discrete time linear Gaussian system over AWGN channel. In [10], the authors addressed the continuous time version of the problem addressed in [9] by focusing on the stability of a linear system with distinct real eigenvalues over SISI AWGN channel. In [16], the authors considered a framework for discussing control over a communication channel based on Signal-to-Noise Ratio (SNR) constraints and focused particularly on the feedback stabilization of an open loop unstable plant via a channel with a SNR constraint. By examining the simple case of a linear time invariant plant and an AWGN channel, they derived necessary and sufficient conditions on the SNR for feedback stabilization with an LTI controller. In [17], the authors presented a sub-optimal decentralized control technique for bounded mean square stability of a large scale system with cascaded clusters of sub-systems. Each sub-system is linear and time-invariant and both sub-system and its measurement are subject to Gaussian noise. The control signals are exchanged between sub-systems without any imperfections, but the measurements are exchanged via an AWGN communication network. In [20], the authors investigated stabilization and performance issues for MIMO LTI networked feedback systems, in which the MIMO communication link is modeled as a parallel noisy AWN channel.

The above literature review reveals that the available results for the stability and tracking of dynamic systems over AWGN channel are concerned with linear dynamic systems and (to the best of our knowledge) there is no result on the stability and reference tracking of nonlinear dynamic systems over AWGN channel. Nevertheless, many networked control systems, such as tele-operation system of autonomous vehicles [1] and smart oil drilling system [2] includes nonlinear dynamic systems. This motivates research on the stability and reference tracking problems of nonlinear dynamic systems over AWGN channel.

As nonlinear analysis and design takes an important role in system analysis and design, several methods are available in the literature, including perturbation method, averaging method, harmonic balance method, etc. [23]-[25] for nonlinear analysis and design. Nonlinear analysis can also be conducted in the frequency domain, using for example, the descending function method [26] and the Volterra series theory [27]-[29]. Using the Volterra series expansion, the Generalized Frequency Response Function (GFRF) was defined in [30], which is a multivariate Fourier transform of the Volterra kernels. This provides a useful concept for nonlinear analysis in the frequency domain, which generalizes the transfer function concept from linear systems to nonlinear systems. Moreover, a systematic method for nonlinear analysis, design, and estimation in the frequency domain is nonlinear Characteristic Output Spectrum (nCOS). The nCOS function is an analytical and explicit expression for the relationship between nonlinear output spectrum and system characteristic parameters and can provide a significant insight into nonlinear analysis and design in the frequency domain ([27], [31] and [32]).

B. Paper Contributions

In this paper, a new technique for reference tracking and stability of nonlinear dynamic systems over AWGN channel, as shown in Fig. 1, is presented. To achieve this goal, using the describing function method, for a nonlinear dynamic system that has periodic outputs to sinusoidal inputs and is cascaded with a bandpass filter acting as encoder, the equivalent linear dynamic system is extracted. Then, we extend the result of [10] to account for reference tracking of linear continuous time systems with multiple real valued and non-real valued eigenvalues over Multi-Input, Multi-Output (MIMO) AWGN channel. Then, by applying these extended results on the equivalent linear dynamic system, we address mean square asymptotic reference tracking (and hence stability) of the nonlinear dynamic system over MIMO AWGN channel, as is shown in the block diagram of Fig. 1. Similar block diagram that are concerned with communication imperfections only from system to remote controller has been considered in many research papers, such as [6], [8]-[13]. This block diagram can correspond to the smart oil drilling system that uses down hole telemetry [2] and also tele-operation systems of micro autonomous vehicles [1]. In the block diagram of Fig. 1, the encoder is a bandpass filter cascaded with a matrix gain; and hence, the nonlinear dynamic system has a describing function as it has periodic outputs to sinusoidal inputs (e.g., the unicycle dynamic model [1], which can represent the dynamic

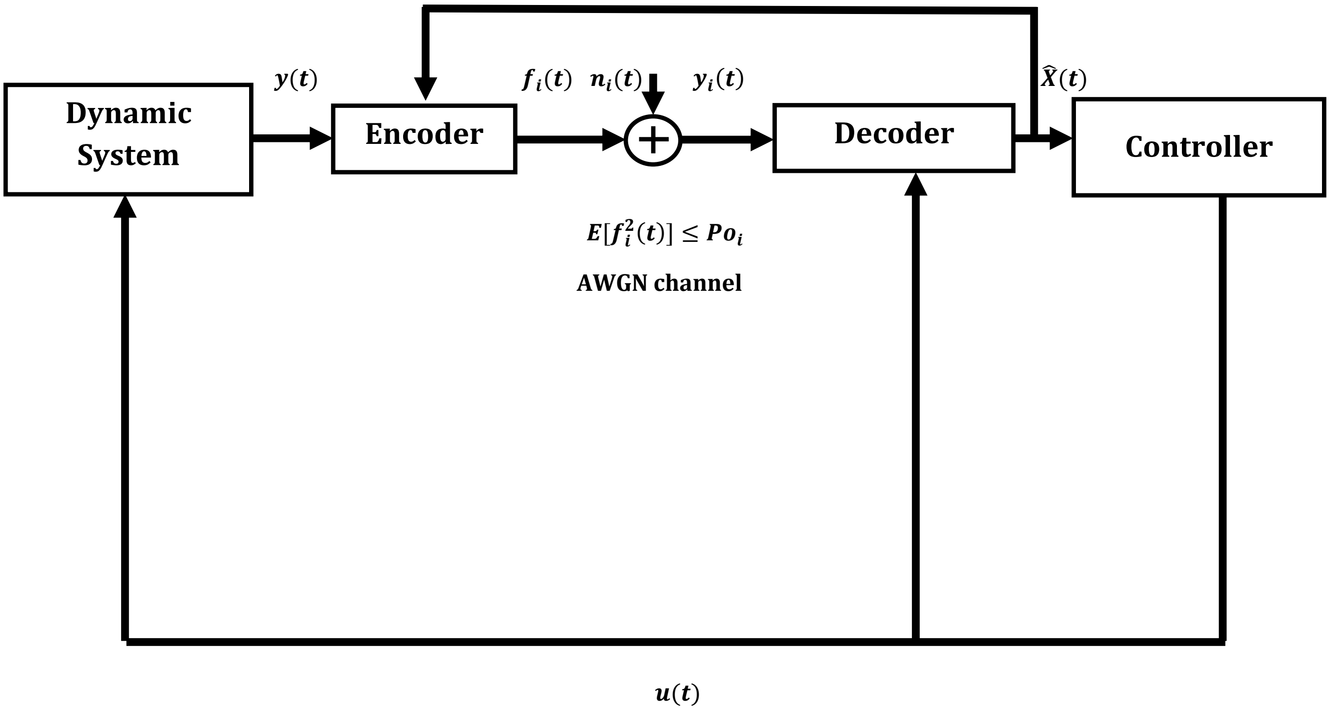

Fig. 1. A dynamic system over MIMO AWGN channel.

of autonomous vehicles). That is, it can be represented by a linear dynamic system. In the field of nonlinear dynamic systems, the describing function is used for building oscillators [26]. However, in this paper, it is used to stabilize and control the nonlinear dynamic system in the block diagram of Fig. 1. This is achieved in this paper by designing an encoder, decoder and a controller for the equivalent linear dynamic system that result in asymptotic mean square tracking of the reference signal.

C. Paper Organization

The paper is organized as follows. In Section II, the problem formulation is presented. Section III is devoted to the describing function method. Then, in Section IV the theory of mean square reference tracking of linear dynamic systems with multiple real valued and non-real valued eigenvalues over MIMO AWGN channel is developed. Section V is devoted to mean square reference tracking of the nonlinear control system of Fig. 1 over MIMO AWGN channel. Simulation results are given in Section VI and the paper is concluded by summarizing the contributions of the paper and directions for future research in Section VII.

II. Problem Formulation

Throughout, certain conventions are used: $E[\cdot]$ denotes the expected value, $|\cdot|$ the absolute value and V' the transpose of vector/matrix V. A^{-1} denotes the inverse of a square matrix A and $N(m,n)$ the Gaussian distribution with mean m and covariance n. $\mathbb{R}$ denotes the set of real numbers and I_n the identity matrix with dimension n by n. $trac(A)$ denotes the trace of a square matrix A, $diag\{.\}$ denotes the diagonal matrix, $[A]_{ij}$ denotes the i,jth element of the matrix A and $\underline{0}$ denotes the zero vector/matrix.

This paper is concerned with asymptotic mean square stability and reference tracking of nonlinear dynamic systems over AWGN communication channel, as is shown in the block diagram of Fig. 1. The building blocks of Fig. 1 are described below.

Dynamic System: The dynamic system is SISO described by the following nonlinear differential equation:

$$\begin{cases} f(t, x^{(m)}, x^{(m-1)}, ..., x, u^{(m)}, u^{(m-1)}, ..., u) = 0 \\ y(t) = x(t) \end{cases} \tag{1}$$

where $x^{(m)}(t) \in \mathbb{R}$ is the mth derivative of the system output, $u(t) \in \mathbb{R}$ is the control input signal, $u^{(m)}(t) \in \mathbb{R}$ is the mth derivative of the control input signal and $f(t, x^{(m)}, x^{(m-1)}, ..., x, u^{(m)}, u^{(m-1)}, ..., u) = 0$ is any arbitrary nonlinear differential equation describing the dynamics of the nonlinear system with the condition that it has periodic outputs in response to sinusoidal inputs.

Communication Channel: Communication channel between system and controller is a MIMO AWGN channel without interference with n inputs and n outputs. The output of the encoder (which will be described shortly) is transmitted through the MIMO channel and a white Gaussian noise vector is added to it (as is shown in Fig. 1), where $N(t) = \begin{bmatrix} n_1(t) & . & . & . & n_n(t) \end{bmatrix}'$ $i.i.d. \sim N(\underline{0}, \tilde{R})$ ($\tilde{R} = diag\{\tilde{r}_1, ..., \tilde{r}_n\}$; $\tilde{r}_i$ is the variance of the additive noise of the ith input to output path of the channel) is the MIMO channel noise and $n_i(t) \sim N(0, \tilde{r}_i)$ is the additive noise of the ith input to output path of the MIMO AWGN channel. Also, the MIMO AWGN channel is subject to the channel input power constraint (Po_i, $i = 1, 2, ..., n$) as follows: $E[f_i^2(t)] \leq Po_i$, $i = 1, 2, ..., n$, where $f_i(t)$ is the ith element of the encoder output vector $F(t) = \begin{bmatrix} f_1(t) & . & . & . & f_n(t) \end{bmatrix}'$, that is the input of the channel. Thus, $Y(t) = F(t) + N(t)$, where $Y(t) = \begin{bmatrix} y_1(t) & . & . & . & y_n(t) \end{bmatrix}'$ is the channel output.

Encoder: The encoder is a bandpass filter cascaded with a matrix gain. This bandpass filter saves only the fundamental frequency of the system output $y(t)$ and omits the other harmonics which have less information to be sent. For this purpose, a high pass filter with a relatively low cut off frequency (e.g., 0.1 Hz) is used to omit the DC part. This filter is cascaded with a low pass filter with a cut off frequency of $\omega \gg 2\pi \times 0.1$ rad/s. For constructing such a filter we can use the following transfer functions [33]:

$$H_{bp}(s) = H_{hp}(s)H_{lp}(s), \tag{2}$$

$$H_{hp}(s) = \frac{s^2}{s^2 + \frac{\omega_h}{Q}s + \omega_h^2}, \omega_h = 2\pi \times 0.1 \ rad/s, \ Q = damping \ factor = 0.707, \tag{3}$$

$$H_{lp}(s) = \frac{\omega^2}{s^2 + \frac{\omega}{Q}s + \omega^2}. \tag{4}$$

It will be shown in the next section that the nonlinear dynamic system (1) together with the band-pass filter has an equivalent linear dynamic system with n states $X(t) = \begin{bmatrix} x_1(t) & . & . & . & x_n(t) \end{bmatrix}'$, in which these states and the matrix gain $C(t) = \begin{bmatrix} C_{11}(t) & C_{12}(t) & ... & C_{1n}(t) \\ C_{21}(t) & C_{22}(t) & ... & C_{2n}(t) \\ & & ... & \\ C_{n1}(t) & C_{n2}(t) & ... & C_{nn}(t) \end{bmatrix}$ form the encoder output $F(t) = C(t)(X(t) - \hat{X}(t))$, where $\hat{X}(t) = \begin{bmatrix} \hat{x}_1(t) & . & . & . & \hat{x}_n(t) \end{bmatrix}'$ is the estimation of states at decoder.

Decoder: Decoder is the minimum mean square estimator or the Kalman filter, also known as Linear Quadratic Estimator (LQE) [34]. At each time instant t, the Kalman filter generates an estimation $\hat{X}(t)$ using the channel output $Y(t)$.

Controller: Controller is a certainty equivalent controller [34] of the following form: $u(t) = -L(t)\hat{X}(t) + \nu(t)$.

The objective of this paper is to find the matrix gains $C(t)$, $L(t)$ and $\nu(t)$ to force the states of the linear equivalent system mean square asymptotically track the reference signal $R(t) = \begin{bmatrix} r_1(t) & . & . & . & r_n(t) \end{bmatrix}'$ (where r_i is the reference signal of the ith state of the linear equivalent system). Note that one of the state of the equivalent linear system is the nonlinear system output $y(t)$; and therefore, the corresponding reference signal for this state must be $r(t)$, which is the reference signal of the dynamic system output $y(t)$. Note also that the stability is a special case of reference tracking with $R(t) = \underline{0}$. Hence, the proposed technique in this paper causes the nonlinear system output $y(t)$ to mean square asymptotically track the reference signal $r(t)$. Now, the exact definition for the reference tracking is given below.

Definition 2.1: (Mean Square Asymptotic Tracking of Reference Signal). Consider the block diagram of Fig. 1 described by the nonlinear dynamic system (1) over AWGN channel, as described earlier with the equivalent linear dynamic system with states $X(t)$. It is said that the system is mean square asymptotically track the reference signal $R(t)$ (and consequently $y(t)$ tracks $r(t)$) if there exist an encoder, decoder and a controller such that the following property holds for all choices of the initial conditions in spite of the noise and the channel input power constraint.

$$\lim_{t\to\infty} E[(x_i(t)-r_i(t))^2]=0,$$
$$subject\ to\ \ E[f_i^2(t)] \leq Po_i,\quad \forall i. \tag{5}$$

III. Implementation of Describing Function Method

In this section, we use the idea of describing function to obtain the equivalent linear dynamic system for the nonlinear dynamic system (1). Then, in the next section we propose an encoder, decoder and a controller for mean square asymptotic tracking of the equivalent linear dynamic system, as defined in this section.

For all nonlinear dynamic systems that respond periodically to sinusoidal inputs, we can find an equivalent linear dynamic system, as defined below [26]:

Consider a SISO nonlinear dynamic system with periodic outputs in response to sinusoidal inputs (e.g., the system (1)). Suppose that this nonlinear dynamic system is excited by the following input: $u(t)=\gamma\cos(\omega t)$, where $\gamma>0$ is large enough to excite all modes of the nonlinear system and ω is the high cut off frequency of the filter. Then, as the output is a periodic signal, it has a Fourier series representation [35] that includes all harmonics of the input with frequency of ω. That is

$$\begin{aligned} y(t) &= y_d+\sum_{i=1}^{\infty}(a_i\sin(i\omega t)+b_i\cos(i\omega t)), \\ y_d &= \frac{\omega}{4\pi}\int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} y(t)dt, \\ a_i &= \frac{\omega}{2\pi}\int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} (y(t)\sin(i\omega t))dt \\ b_i &= \frac{\omega}{2\pi}\int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} (y(t)\cos(i\omega t))dt. \end{aligned} \tag{6}$$

Now, if this nonlinear system is cascaded with a bandpass filter with high cut-off frequency ω, then we have a periodic output at the end of the filter which consists of only the first harmonic with frequency ω, and all other harmonics are eliminated. In other words, the output of the bandpass filter is the following:

$$y_f(t) = a_1 \sin(\omega t) + b_1 \cos(\omega t). \tag{7}$$

Having that, we call the nonlinear dynamic system that is cascaded with the bandpass filter with the high cut off frequency of ω, a quasi linear system [26]. Because, we can find a linear dynamic system with the input $u(t) = \gamma \cos(\omega t)$ and the output $y_f(t)$ with the following transfer function:

$$H(j\omega) = |H(j\omega)| \angle H(j\omega). \tag{8}$$

$$|H(j\omega)| = \frac{\sqrt{a_1^2 + b_1^2}}{\gamma}. \tag{9}$$

$$\angle H(j\omega) = -\arctan(\frac{a_1}{b_1})\ rad. \tag{10}$$

This means that the nonlinear dynamic system can be represented by a linear dynamic system with the above transfer function called the describing function of the nonlinear dynamic system. Note that the describing function in s domain can be obtained by assuming the following form for the describing function $H(s) = \frac{1}{s^2+\alpha s+\beta}$ (usually second order transfer function is enough to show the behavior of the fundamental frequency of the dynamic system); and then determining the real coefficients α and β such that $|H(j\omega)| = \frac{\sqrt{a_1^2+b_1^2}}{\gamma}$ and $\angle H(j\omega) = -\arctan(\frac{a_1}{b_1})$ rad. This describing function represents a linear system with $n = 2$ states: $x_1(t)$, $x_2(t)$ (e.g., $x_1(t) = y(t)$ and $x_2(t) = \dot{y}(t)$ with realization $A = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$), one input $u(t)$ and two outputs represented by vector $X(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}'$, which is input to the encoder matrix gain $C(t)$.

For a linear dynamic system with realization (A, B) where the A matrix has only real valued eigenvalues, a mean square stabilizing technique over SISO AWGN channel is presented in [10] by finding the matrix gain $C(t)$ and $L(t)$. In the next section, we first extend this result to account for MIMO AWGN channel with a system matrix A with multiple real valued and non-real valued eigenvalues. Then, by applying this extension on the linear equivalent dynamic system, we present the result for mean square asymptotic tracking of the nonlinear dynamic system (1) with the equivalent linear dynamic system (8) over MIMO AWGN channel.

IV. Linear System with Multiple Real Valued and Non-real Valued Eigenvalues

Now, we extend the result of [10] to account for reference tracking and hence stability of linear continuous time dynamic systems with multiple real valued and non-real valued eigenvalues over MIMO AWGN channel.

Suppose that the dynamic system in Fig. 1 is linear with n states. Hence, the system that is seen by the remote controller is as follows:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t), \ X(0) = \xi \\ Y(t) = C(t)(X(t) - \hat{X}(t)) + N(t) \end{cases} \tag{11}$$

where $X(0)$ is the initial state, which is unknown for remote controller, $\xi \sim N(X_0, Q_0)$ (Q_0 is diagonal) is a random variable and $N(t) = \begin{bmatrix} n_1(t) & . & . & . & n_n(t) \end{bmatrix}'$ $i.i.d. \sim N(\underline{0}, \tilde{R})$ ($\tilde{R} = diag\{\tilde{r}_1, ..., \tilde{r}_n\}$) is the additive noise of the MIMO channel ($n_i(t) \sim N(0, \tilde{r}_i)$ is the additive noise of the ith path of the MIMO AWGN channel) and can be treated as the measurement noise provided the channel input power constraint is met. The first objective in this section is to achieve mean square asymptotic state tracking, i.e., $E[(x_i(t) - \hat{x}_i(t))^2] \to 0$, $\forall i$. In this section, we deal with the linear system (11) with n states over MIMO AWGN channel. As discussed in the previous section, this system can represent the state space realization of the transfer function $H(s)$ (e.g., $A = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$), which, in general, includes multiple real valued eigenvalues and non - real valued eigenvalues.

Throughout, it is assumed that the system matrix A has real eigenvalues, real multiple eigenvalues and distinct complex conjugate eigenvalues as the system matrix A that corresponds to the transfer function $H(s)$ includes these types of eigenvalues. Without loss of generality, we can always assume that the system matrix A is in the real Jordan form [8]. This form is obtained by implementing a proper similarity transformation [36]; and as a result of that, the system (11) is decomposed to several decoupled sub-systems, in which for each sub-system, an encoder and a decoder can be designed separately for the state tracking. Note that the Jordan block associated with a real eigenvalue $\lambda_i(A)$ with multiplicity d_i is the following matrix

$$\begin{bmatrix} \lambda_i(A) & a & 0 & . & . & . & 0 & 0 \\ 0 & \lambda_i(A) & b & . & . & . & 0 & 0 \\ . & & & & & & & \\ . & & & & & & & \\ . & & & & & & & \\ 0 & 0 & 0 & . & . & . & \lambda_i(A) & c \\ 0 & 0 & 0 & . & . & . & 0 & \lambda_i(A) \end{bmatrix}$$

where $a, b, c \in \{0, 1\}$ depending on the rank of matrix $(\lambda_i(A)I_n - A)$. Note also that the Jordan block associated with the complex conjugate pair of the eigenvalues $\lambda_i(A) = \sigma \pm \sqrt{-1}\omega$ $(\omega \neq 0)$ is $\begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$.

A. Mean square asymptotic state tracking

In this section, it is assumed that each of the Jordan block is at most a 2 by 2 matrix (the general case can be treated similarly). Then, for all three possible cases: a) $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, $a_1, a_2 \in \mathbb{R}, a_1 \neq a_2$, b) $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, $a \in \mathbb{R}$, c) $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$, $\sigma, \omega \in \mathbb{R}, \omega \neq 0$; we find the matrix gain $C(t)$ for mean square asymptotic tracking of system states at the decoder.

1) Sub-system with real distinct eigenvalues: Suppose that the system matrix A in the linear system of (11) has the following form

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, a_1, a_2 \in \mathbb{R},\ a_1 \neq a_2. \tag{12}$$

Then, we have the following proposition for mean square asymptotic tracking of system states at the decoder.

Proposition 4.1: Consider the block diagram of Fig. 1 described by the linear system (11) with the system matrix $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, $a_1, a_2 \in \mathbb{R}, a_1 \neq a_2$. Suppose that the pair (A, B) is controllable and $Po_1 > 2a_1$ and $Po_2 > 2a_2$ (Po_1 and Po_2 are the channel input power constraints). Then, by choosing the encoder matrix gain, as follows: $C(t) = \begin{bmatrix} \sqrt{\frac{Po_1 \tilde{r}_1}{P_{11}(t)}} & 0 \\ 0 & \sqrt{\frac{Po_2 \tilde{r}_2}{P_{22}(t)}} \end{bmatrix}$, and a decoder with the following description (which is the Kalman filter [34])

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0,$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1}, \ \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\}$$

$$\dot{P}(t) = A'P(t)+P(t)A-P(t)C'(t)\tilde{R}^{-1}C(t)P(t), \ P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) \\ P_{12}(t) & P_{22}(t) \end{bmatrix} \geq 0$$

we have mean square asymptotic tracking of system states at the decoder.

Proof: The encoder matrix gain has the following general form $C(t) = \begin{bmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{bmatrix}$; and subsequently, the decoder can be extracted as follows (for the simplicity of presentation the dependency to the time index t is dropped):

$$\begin{cases} \dot{P}_{11} = 2a_1P_{11} - (2P_{11}P_{12}C_{11}C_{12}\tilde{r}_1^{-1} + 2P_{11}P_{12}C_{21}C_{22}\tilde{r}_2^{-1} + P_{11}^2C_{11}^2\tilde{r}_1^{-1} + P_{12}^2C_{12}^2\tilde{r}_1^{-1} \\ \qquad +P_{11}^2C_{21}^2\tilde{r}_2^{-1} + P_{12}^2C_{22}^2\tilde{r}_2^{-1}) \\ \dot{P}_{22} = 2a_2P_{22} - (2P_{22}P_{12}C_{22}C_{21}\tilde{r}_2^{-1} + 2P_{22}P_{12}C_{21}C_{11}\tilde{r}_1^{-1} + P_{22}^2C_{22}^2\tilde{r}_2^{-1} + P_{12}^2C_{21}^2\tilde{r}_2^{-1} \\ \qquad +P_{22}^2C_{12}^2\tilde{r}_1^{-1} + P_{12}^2C_{11}^2\tilde{r}_1^{-1}) \\ \dot{P}_{12} = a_1P_{12} + a_2P_{12} - \tilde{r}_1^{-1}(P_{11}P_{12}C_{11}^2 + P_{12}^2C_{11}C_{12} + P_{11}P_{22}C_{11}C_{12} + P_{12}P_{22}C_{12}^2) \\ \qquad -\tilde{r}_2^{-1}(P_{11}P_{12}C_{21}^2 + P_{12}^2C_{22}C_{21} + P_{11}P_{22}C_{22}C_{21} + P_{12}P_{22}C_{22}^2) \end{cases} \tag{13}$$

Now, by substituting $C_{11}(t) = \sqrt{\frac{Po_1\tilde{r}_1}{P_{11}(t)}}$, $C_{22}(t) = \sqrt{\frac{Po_2\tilde{r}_2}{P_{22}(t)}}$ and $C_{12}(t) = C_{21}(t) = 0$ in the above expressions, we have $\dot{P}_{12}(t) = 0$ and $P_{12}(t) = P_{12}(0) = [Q_0]_{12} = 0$. Thus, we have

$$\dot{P}_{11}(t) = (2a_1 - Po_1)P_{11}(t) \Rightarrow P_{11}(t) = e^{-(Po_1-2a_1)t}P_{11}(0). \tag{14}$$

$$\dot{P}_{22}(t) = (2a_2 - Po_2)P_{22}(t) \Rightarrow P_{22}(t) = e^{-(Po_2-2a_2)t}P_{22}(0). \tag{15}$$

In summary, by the above selection, we have

$$P(t) = \begin{bmatrix} e^{-(Po_1-2a_1)t}P_{11}(0) & 0 \\ 0 & e^{-(Po_2-2a_2)t}P_{22}(0) \end{bmatrix} \tag{16}$$

and the following description for the decoder:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \ K(t) = P(t)C^{'}(t)\tilde{R}^{-1}, \quad \hat{X}(0) = X_0. \tag{17}$$

Now, as we assumed that $Po_1 > 2a_1$ and $Po_2 > 2a_2$, we have $\lim_{t\to\infty} E[(x_1(t) - \hat{x}_1(t))^2] = \lim_{t\to\infty} P_{11}(t) = \lim_{t\to\infty} e^{-(Po_1-2a_1)t}P_{11}(0) = 0$. Similarly, we have $\lim_{t\to\infty} E[(x_2(t)-\hat{x}_2(t))^2] = \lim_{t\to\infty} P_{22}(t) = 0$. Hence, $P(t) \to \bar{P} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. This completes the proof.

2) Sub-system with real multiple eigenvalues: Suppose that the system matrix A in the linear system of (11) has the following form

$$A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, a \in \mathbb{R}. \tag{18}$$

Then, we have the following proposition for mean square asymptotic tracking of system states at the decoder.

Proposition 4.2: Consider the block diagram of Fig. 1 described by the linear system (11) with the system matrix $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, $a \in \mathbb{R}$. Suppose that the pair (A, B) is controllable, $Po_1 > 2a$ and $Po_2 > 2a$, $[Q_0]_{11} = [Q_0]_{22}$ and $\tilde{r}_1 = \tilde{r}_2$. Then, by choosing the encoder matrix gain as $C(t) = \begin{bmatrix} \sqrt{\frac{1}{2\delta P_{22}(t)}} & \sqrt{\frac{\delta}{2P_{22}(t)}} \\ \sqrt{\frac{\delta}{2P_{22}(t)}} & \sqrt{\frac{1}{2\delta P_{22}(t)}} \end{bmatrix}$, where $\delta = \gamma_1 - \sqrt{\gamma_1^2 - 1}$ and $\gamma_1 = \min(Po_1, Po_2)$, and the following decoder (which is the Kalman filter [34])

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0,$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1}, \; \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\}$$

$$\dot{P}(t) = A'P(t) + P(t)A - P(t)C'(t)\tilde{R}^{-1}C(t)P(t), \; P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) \\ P_{12}(t) & P_{22}(t) \end{bmatrix} \geq 0$$

we have mean square asymptotic tracking of system states at the decoder.

Proof: It is similar to the proof of Proposition 4.1.

3) Sub-system with complex conjugate eigenvalues: Suppose that the system matrix A in the linear system of (11) has the following form

$$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}, \sigma, \omega \in \mathbb{R}, \omega \neq 0. \tag{19}$$

Then, we have the following proposition for mean square asymptotic tracking of system states at the decoder.

Proposition 4.3: Consider the block diagram of Fig. 1 described by the linear system (11) with the system matrix $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$, $\sigma, \omega \in \mathbb{R}, \omega \neq 0$. Suppose that the pair (A, B) is controllable, $Po_1 > 2\sigma$ and $Po_2 > 2\sigma$, $[Q_0]_{11} = [Q_0]_{22}$ and $\tilde{r}_1 = \tilde{r}_2$. Then, by choosing the encoder matrix gain as $C(t) = \begin{bmatrix} \sqrt{\frac{Po_1 \tilde{r}_1}{P_{11}(t)}} & 0 \\ 0 & \sqrt{\frac{Po_2 \tilde{r}_2}{P_{22}(t)}} \end{bmatrix}$, and the following decoder (which is the Kalman filter [34])

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + K(t)Y(t), \quad \hat{X}(0) = X_0,$$

$$K(t) = P(t)C'(t)\tilde{R}^{-1},\ \tilde{R} = diag\{\tilde{r}_1, \tilde{r}_2\}$$

$$\dot{P}(t) = A'P(t)+P(t)A-P(t)C'(t)\tilde{R}^{-1}C(t)P(t),\ P(0) = Q_0, P(t) = P'(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) \\ P_{12}(t) & P_{22}(t) \end{bmatrix} \geq 0$$

we have mean square asymptotic tracking of system states at the decoder.

Proof: It is similar to the proof of Proposition 4.1.

B. Asymptotic Reference Tracking

Now, to obtain the control signal for tracking the reference signal $R(t)$, consider the following cost functional

$$J = \lim_{t_1 \to \infty} \frac{1}{t_1} \int_0^{t_1} E[[X(t) - R(t)]'Q[X(t) - R(t)] + \rho u^2(t)]dt,\ Q = Q' \geq 0, \rho > 0.$$

The control signal $u(t)$ is obtained by minimizing the above cost functional subject to the dynamic system (11). From [34] it follows that $u(t) = -L(t)\hat{X}(t) + \nu(t)$, where

$$L(t) = \rho^{-1}B'\tilde{P}(t) \tag{20}$$

$$\nu(t) = -\rho^{-1}B'S(t) \tag{21}$$

and $\tilde{P}(t)$ and $S(t)$ are the result of following equations (Q and ρ are arbitrary):

$$\dot{\tilde{P}}(t) = -\tilde{P}(t)A - A'\tilde{P}(t) - Q + \tilde{P}(t)B\rho^{-1}B'\tilde{P}(t) \tag{22}$$

$$\dot{S}(t) = -[A' - \tilde{P}(t)B\rho^{-1}B']S(t) + QR(t) \tag{23}$$

Under the assumption that the pair (A, B) is controllable, we have $\tilde{P}(t) \to \bar{\tilde{P}}$ [34]; and hence, $J \to \bar{J} = 0$. This indicates that $E[X(t) - R(t)]'Q[X(t) - R(t)] \to 0$; and hence, $E[(x_i(t) - r_i(t))^2] \to 0$, $\forall i$ (if the Q matrix is diagonal).

V. Reference Tracking of the Nonlinear Dynamic System

In this section, we have the following proposition for the mean square asymptotic reference tracking of the nonlinear dynamic system (1) (with the equivalent linear dynamic system (8)) in the block diagram of Fig. 1.

Proposition 5.1: Consider the control system of Fig. 1. described by the nonlinear dynamic system (1) over MIMO AWGN channel subject to the power constraint $E[f_i^2(t)] \leq Po_i$, as described earlier. Suppose that the state space realization ($A(\omega)$,$B(\omega)$) for the transfer function $H(s)$, is controllable. Then, there exist an encoder, decoder and a controller that force the states

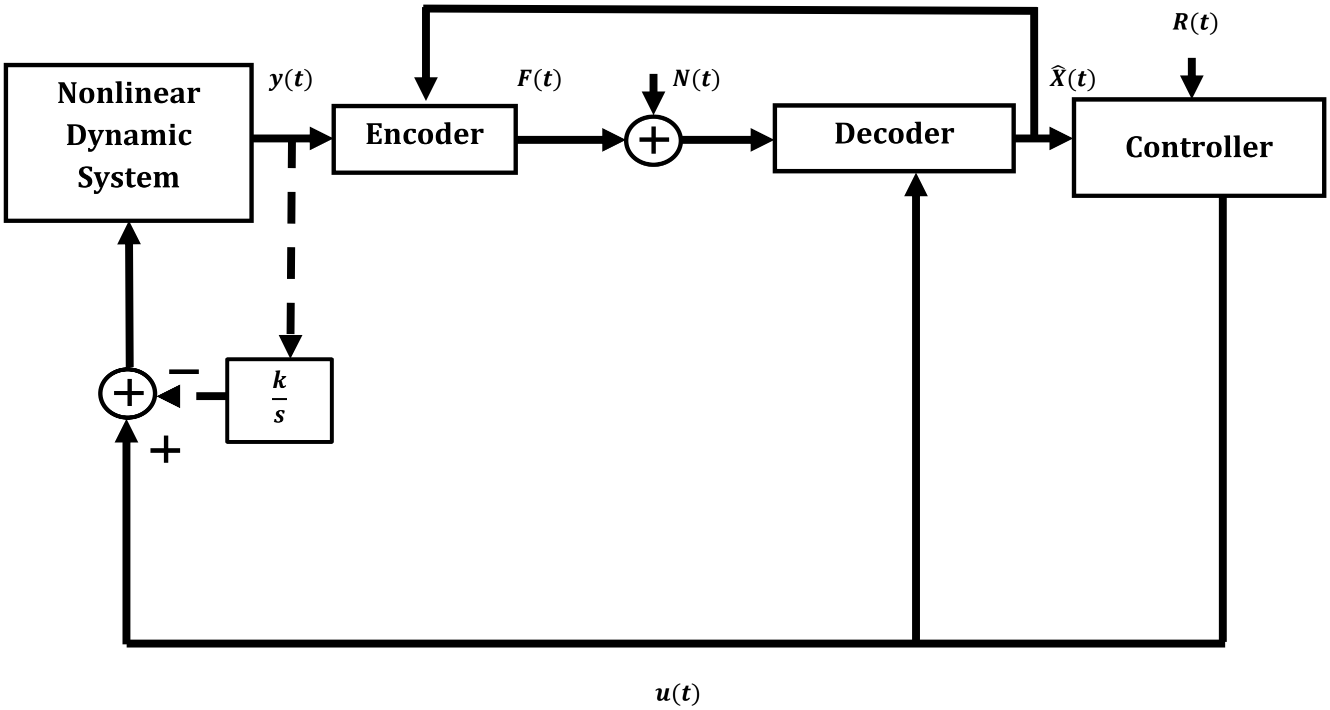

Fig. 2. Nonlinear dynamic system (1) controlled over AWGN channel.

mean square asymptotically track the reference signal $R(t)$; and hence, cause the nonlinear dynamic system output $y(t)$ to mean square asymptotically track the reference signal $r(t)$.

Proof: From Section III it follows that $H(s) = \frac{1}{s^2+\alpha s+\beta}$; and hence, the state space realization for $H(s)$ is ($A(\omega)$,$B(\omega)$) with $n = 2$. Subsequently, the encoder matrix gain $C(t)$ and the decoder are determined from Propositions 4.1-4.3; and hence, from the results of Section IV-B, it follows that the controller of the form of $u(t) = -L(t)\hat{X}(t) + \nu(t)$, where $L(t)$ and $\nu(t)$ are given from (20) and (21), respectively, results in the reference tracking. That is, using this control policy, the output of the bandpass filter minus the reference signal, i.e., $X(t) - R(t)$ converges to zero in mean square sense (see Fig. 2). Note that one of the elements of the vector $X(t)$ (i.e., the state vector of the realization of the describing function) is the system output with the DC part excluded (due to the bandpass filter). Hence, although it is expected that $y(t) - r(t) \to 0$ (as $X(t) - R(t) \to 0$), the designed control signal may make $y(t) - r(t)$ to converge to a non-zero value. Therefore, to make $y(t) - r(t)$ also to converge to zero, we use the inner control loop with gain $\frac{k}{s}$ (where k is large enough, e.g., 25) to attenuate the effect of this DC bias in signal $y(t) - r(t)$ as much as possible so that $y(t)$ also tracks $r(t)$. This completes the proof.

VI. SIMULATIONS RESULTS

In this section, for the purpose of illustration, we apply the proposed encoder, decoder and controller to a DC motor with saturation element, that is used in our laboratory to protect the DC motor from high input voltage as is shown in Fig. 3. Systems with actuator saturation and hard constraints are important class of dynamic systems and have been considered in the literature (e.g., [37]-[39]). The DC motor considered in this paper has the following description

$$\begin{cases} \dot{x}(t) = -0.96x(t) + 178.85\tilde{u}(t) \\ y(t) = x(t), \end{cases} \tag{24}$$

where $\tilde{u}(t)$ is the output of the saturation element. The saturation element saturates the control signal $u(t)$ between -10 and +10 volts. That is,

$$\tilde{u}(t) = \begin{cases} 10, & u(t) \geq 10 \\ u(t), & |u(t)| < 10 \\ -10, & u(t) \leq -10 \end{cases} \tag{25}$$

For the remote controller the initial condition is unknown and has the following description $x(0) \sim N(0,1)$. The system must track the reference signal $r(t)$. The MIMO AWGN channel has the following specification: $N(t)\ i.i.d. \sim N(\underline{0}, diag\{1,1\})$, with the following power constraints $Po_1 = 2$ and $Po_2 = 2$ to satisfy the requirements of Propositions 4.1-4.3.

The linear dynamic system (24) can be treated as a low pass filter, which is cascaded with the nonlinear saturation block (see Fig. 3). Hence, this block together with the linear dynamic system (24) has a quasi linear representation described by the describing function. To obtain this describing function for a given ω, e.g., $\omega = 10$, we apply the following sinusoidal input to the system:

$$u(t) = 15\cos(10t). \tag{26}$$

The steady state output of this excitation signal is:

$$y(t) = x(t) = 208.5\cos(10t + 1.67) \quad rad/s, \tag{27}$$

which is shown in Fig. 4. From this input and output, the describing function is $H(s) = \frac{1}{s^2+0.00716s+99.99}$; and therefore, the equivalent linear dynamic system that is seen by the remote

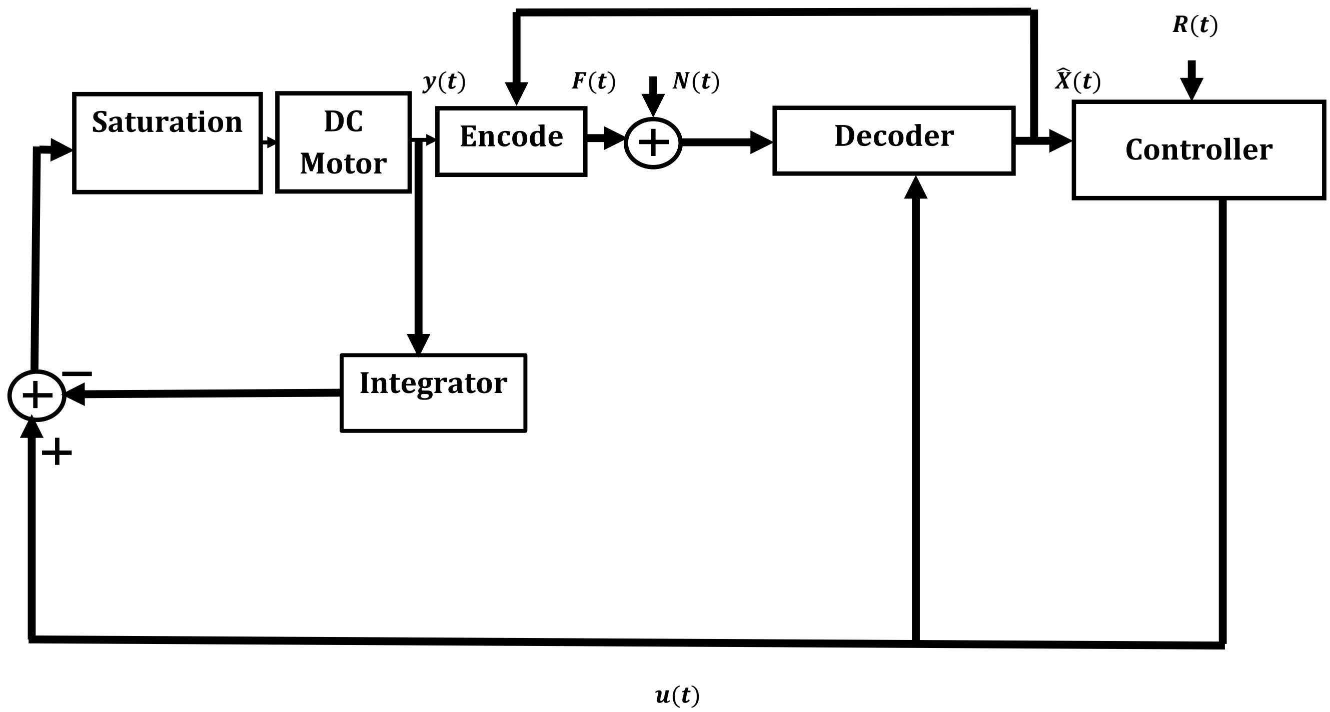

Fig. 3. A nonlinear dynamic system controlled over AWGN channel.

controller in the Jordan form is the following:

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} -0.0036 & -9.9996 \\ 9.9996 & -0.0036 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ -0.1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} X(t) \\ Y(t) = C(t)(X(t) - \hat{X}(t)) + N(t) \end{cases} \tag{28}$$

Note that here $X_1(t) = y(t)$; and therefore, $r_1(t) = r(t)$ Also, the encoder is described by the matrix gain $C(t)$. Now, by applying the previous theoretical developments to this system, we have mean square asymptotic tracking, as follows:

Fig. 5 illustrates the system output $y(t)$ and control signal $u(t)$ for $r(t) = 1000\ rad/s$. Fig. 6 and Fig. 7 illustrate the system output $y(t)$ and control signal $u(t)$ for $r(t) = 1000\cos(0.2t)\ rad/s$ and $r(t) = 2000\cos(0.2t)\ rad/s$, respectively. As is clear from these figures the proposed reference tracking technique is able to track the reference signal with suitable transient response.

To magnify the performance of our proposed technique, we also applied the method of [10], which is for linear system, on the system (24) without considering the saturation in the design of control signal (as the method proposed in [10] is only for linear systems). Fig. 8 illustrates

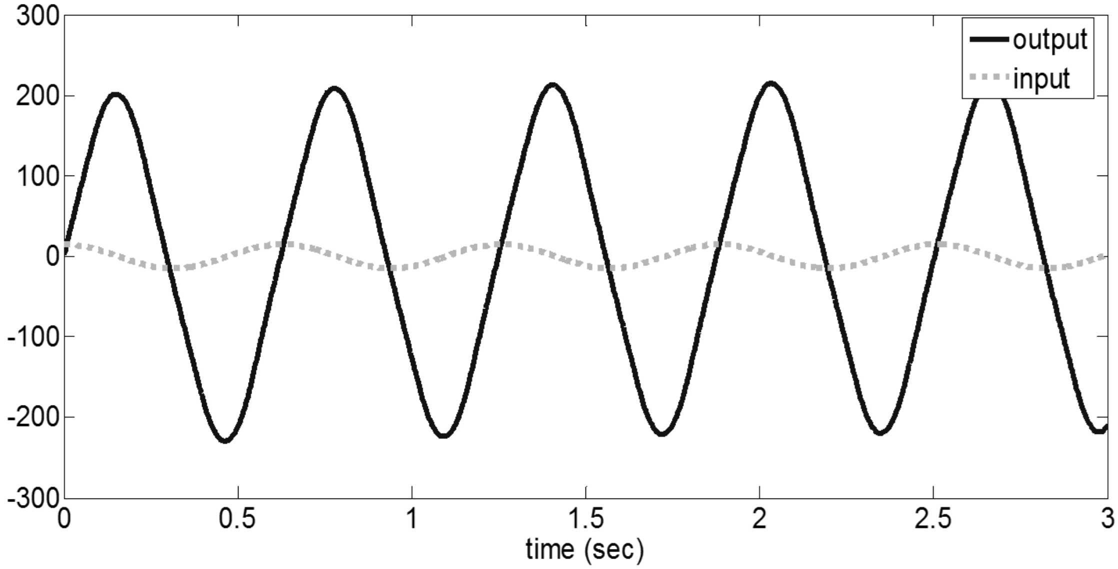

Fig. 4. The output of the nonlinear system for the sinusoidal input $u(t) = 15\cos(10t)$ and $x(0) = 1$.

the simulation result for this case when the reference signal is $r(t) = 2000\cos(0.2t)\ rad/s$. Comparing the result of Fig. 7 with Fig. 8 illustrates that our proposed technique has a better performance.

VII. Conclusion and Direction for Future Research

In this paper, a new technique for mean square asymptotic reference tracking of nonlinear dynamic systems over AWGN channel was presented. To achieve this goal, using the describing function method, for a nonlinear dynamic system that has periodic outputs to sinusoidal inputs and is cascaded with a bandpass filter acting as encoder, an equivalent linear dynamic system was extracted. Then, we extended the results of [10] to account for reference tracking of continuous time systems with multiple real valued and non-real valued eigenvalues over MIMO AWGN channel. Then, by applying these extended results on the equivalent linear dynamic system, a new technique for mean square asymptotic reference tracking of nonlinear dynamic systems over AWGN channel was presented. The proposed technique for mean square asymptotic reference tracking includes an encoder consisting of a bandpass filter cascaded with a matrix gain, the Kalman filter [34] and a certainty equivalent controller [34]. The satisfactory performance of the

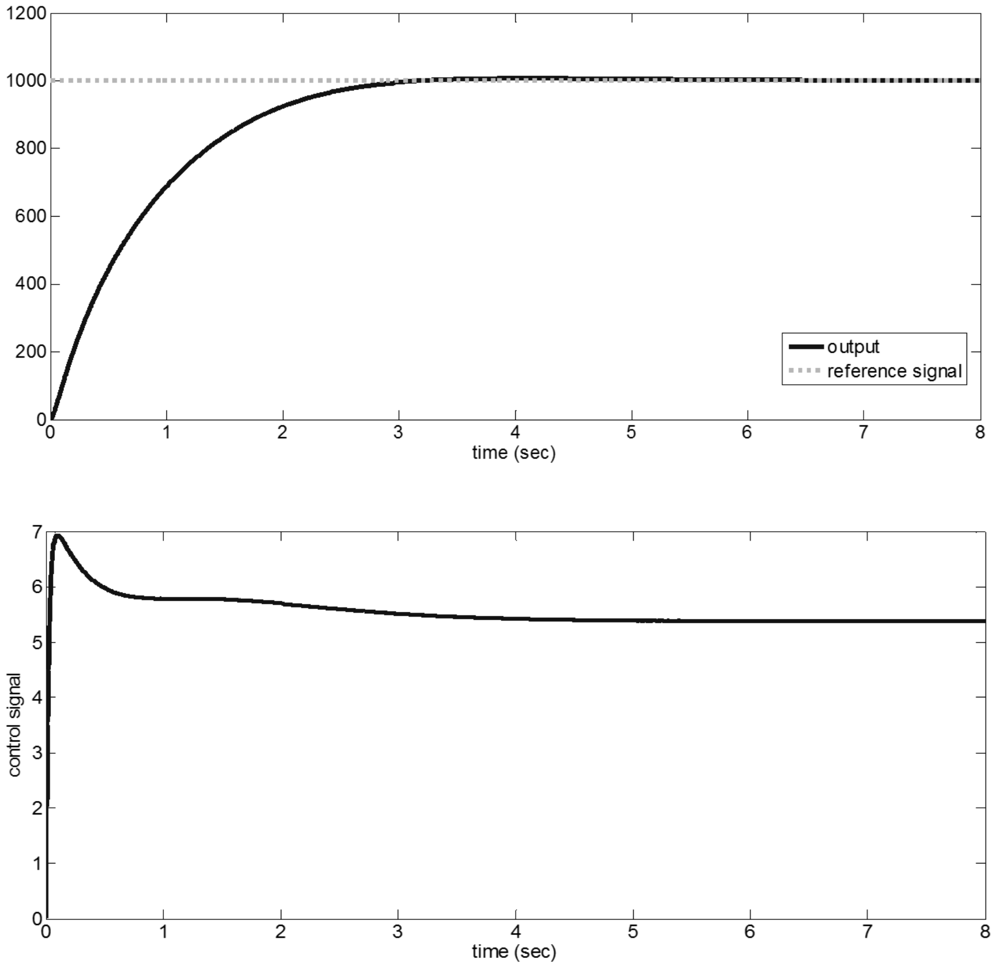

Fig. 5. The response of the system and control signal for $r(t) = 1000\ rad/s$.

proposed technique was illustrated using computer simulations.

For future, it is interesting to find techniques for reference tracking of the other class of nonlinear dynamic systems over AWGN channel. The application of the proposed technique on the nonlinear unicycle model [1], which represents the dynamic of the autonomous vehicles, is currently under investigations by the authors.

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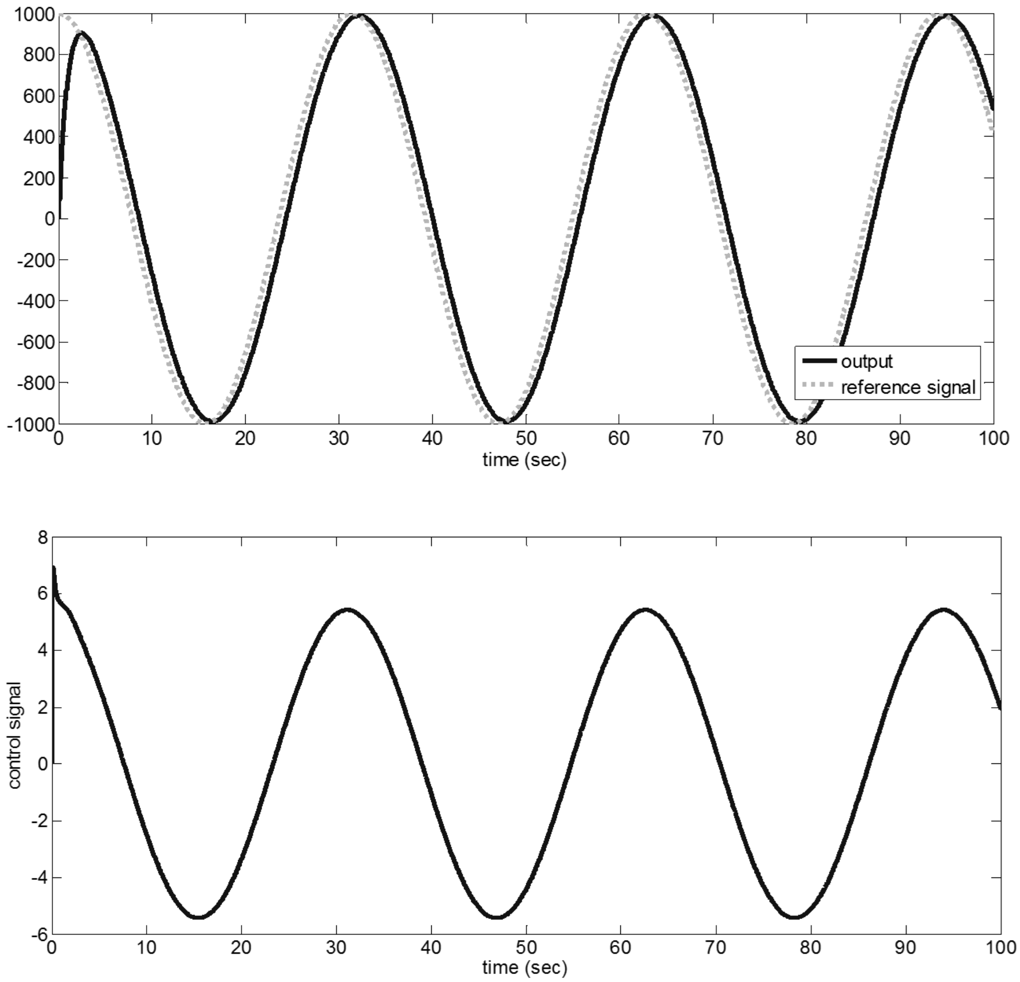

Fig. 6. The response of the system and control signal for $r(t) = 1000\cos(0.2t)\ rad/s$.

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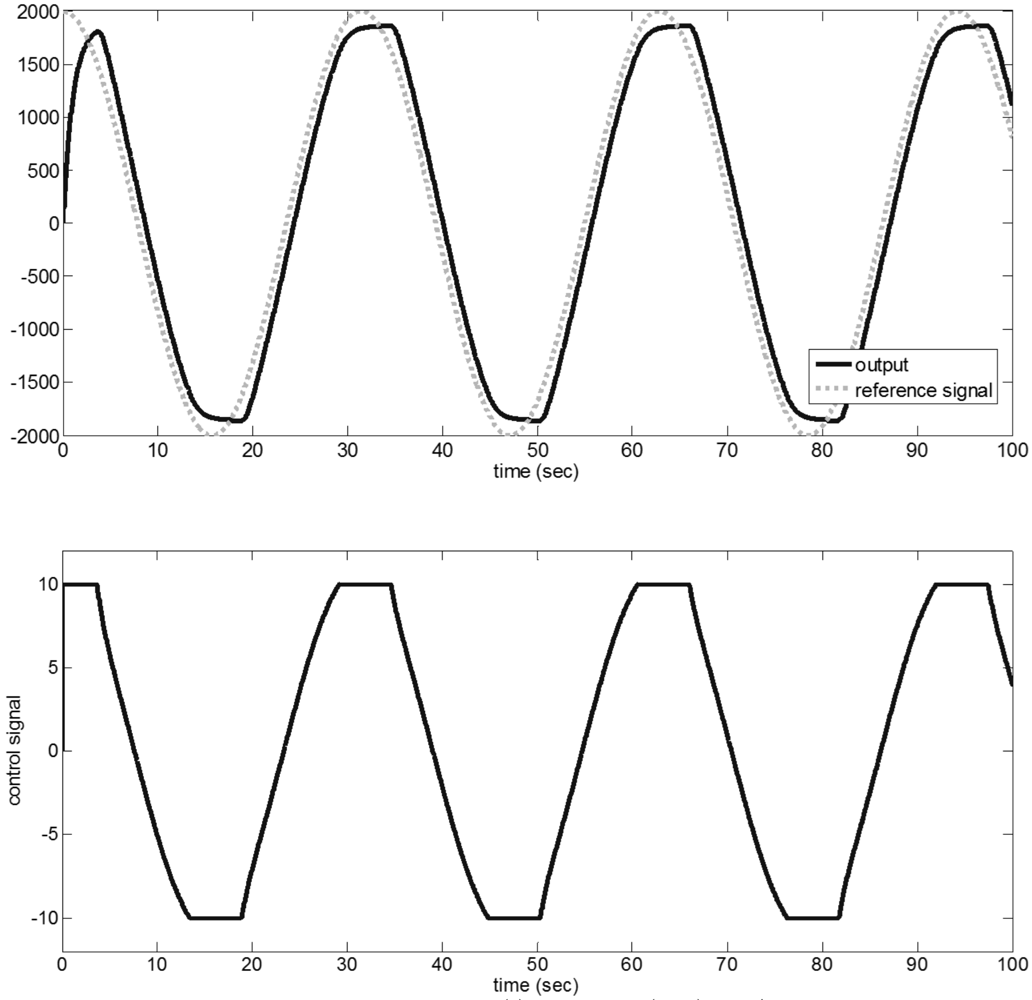

Fig. 7. The response of the system and control signal for $r(t) = 2000\cos(0.2t)\ rad/s$.

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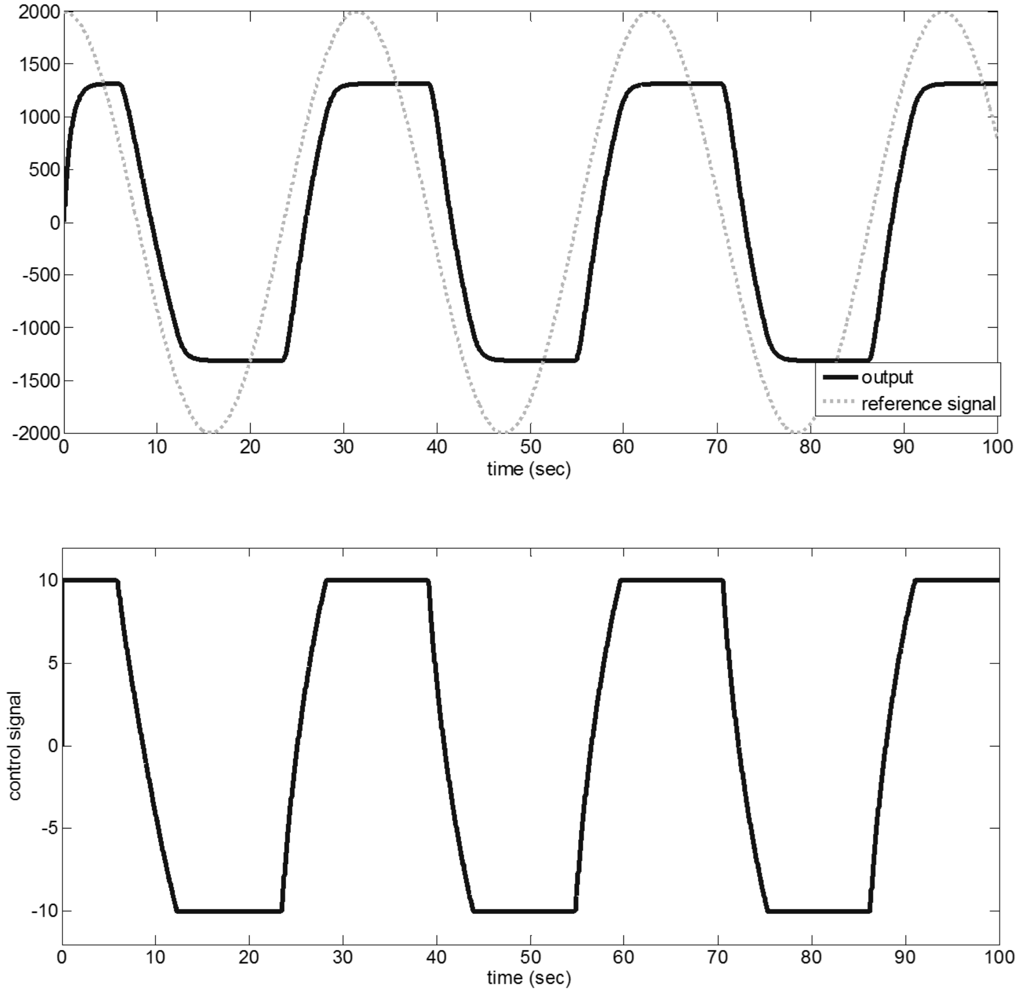

Fig. 8. The response of the system and control signal of the linear method of [10] for $r(t) = 2000\cos(0.2t)\ rad/s$.

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