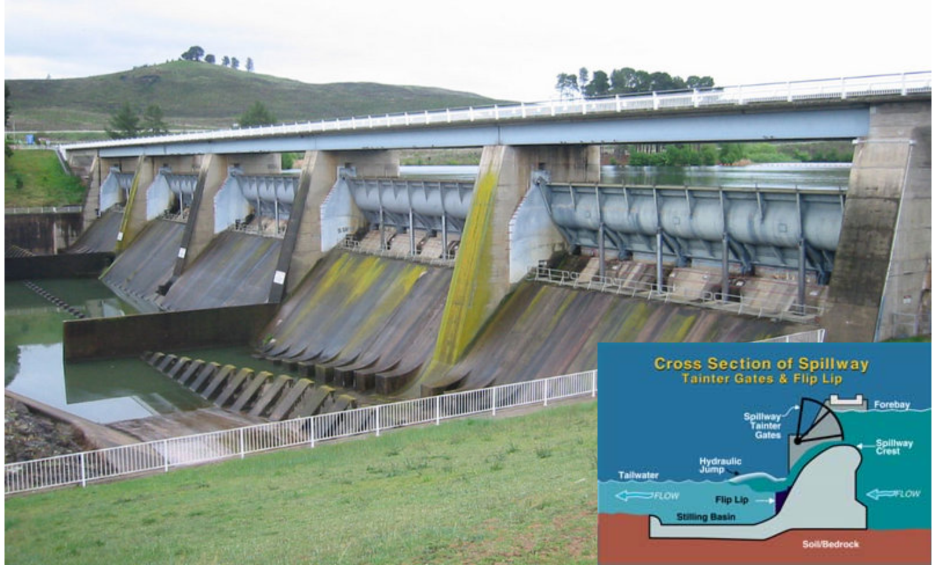
Hydrostatic Forces on Curved Surfaces

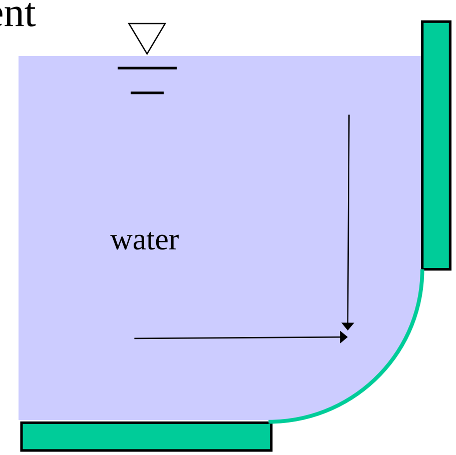


Static Surface Forces

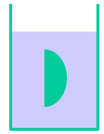
- Forces on plane areas
- Forces on curved surfaces
- Buoyant force
- Stability of floating and submerged bodies

Forces on Curved Surfaces

- Horizontal component
- Vertical component



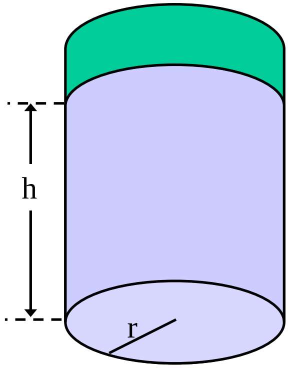
Forces on Curved Surfaces: Horizontal Component



- What is the horizontal component of pressure force on a curved surface equal to? The pressure force on the vertical plane projection
- The center of pressure is located using the moment of inertia or pressure prism technique.
- The horizontal component of pressure force on a closed body is _zero_.

Forces on Curved Surfaces: Vertical Component

- What is the magnitude of the vertical component of force on the cup?



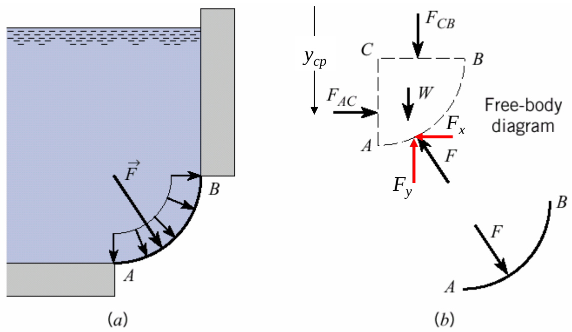
F = pA

p = γh

F = γhπr² =W!

What if the cup had sloping sides?

Pressure on Curved Surface



$$\sum_x F = 0 = F_{AC} - F_x$$

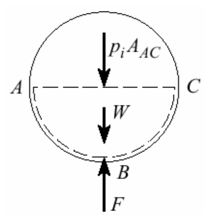
$$F_{AC} = F_x$$

$$\sum_y F = 0 = F_y - F_{CB} - W$$

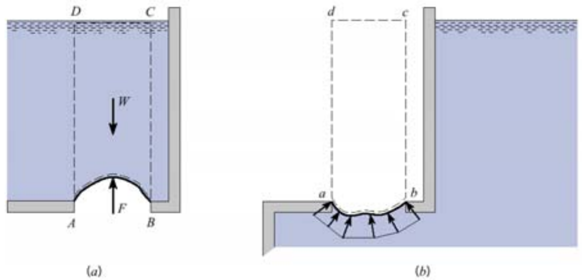
$$F_y = W + F_{CB}$$

Forces on Curved Surfaces: Vertical Component

The vertical component of pressure force on a curved surface is equal to the weight of liquid vertically above the curved surface and extending up to the (virtual or real) free surface.



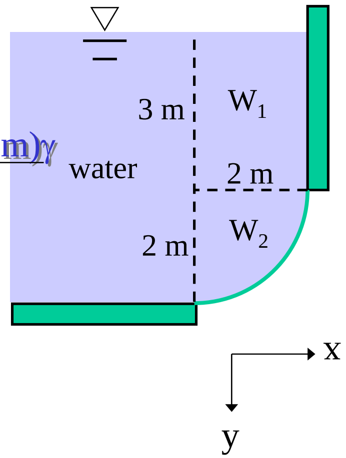
$$F = p_i A_{AC} + W$$



$$F = \gamma V_{ABCD} = W$$

Example: Forces on Curved Surfaces

Find the resultant force (magnitude and location) on a 1 m wide section of the circular arc.



$F_V = W_1 + W_2$

$= (3\text{ m})(2\text{ m})(1\text{ m})\gamma + 1/4\pi(2\text{ m})^2(1\text{ m})\gamma$

$= 58.9\text{ kN} + 30.8\text{ kN}$

$= 89.7\text{ kN}$

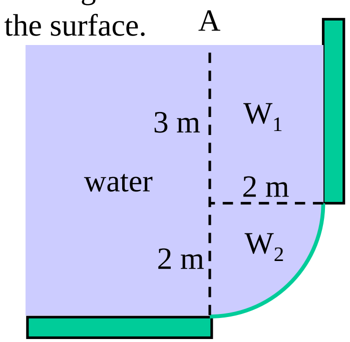
$F_H = p_c A \qquad p_c = \gamma h_c$

$= \gamma(4\text{ m})(2\text{ m})(1\text{ m})$

$= 78.5\text{ kN}$

Example: Forces on Curved Surfaces

The vertical component line of action goes through the centroid of the volume of water above the surface.



Take moments about a vertical axis through A.

$$x_R F_V = (1\text{ m})W_1 + \frac{4(2\text{ m})}{3\pi}W_2 \qquad \frac{4R}{3\pi}$$

$$x_R = \frac{(1\text{ m})(58.9\text{ kN}) + \frac{4(2\text{ m})}{3\pi}(30.8\text{ kN})}{(89.7\text{ kN})}$$

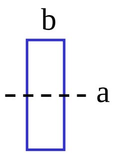
= 0.948 m (measured from A) with magnitude of 89.7 kN

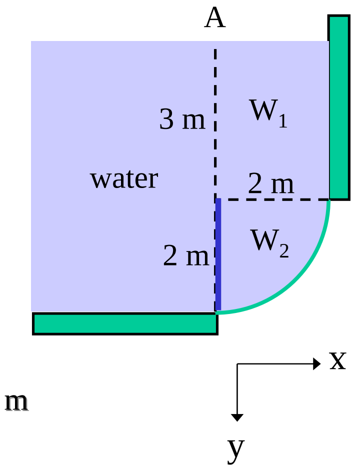
Example: Forces on Curved Surfaces

The location of the line of action of the horizontal component is given by

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$I_{xc} = \frac{ba^3}{12}$$



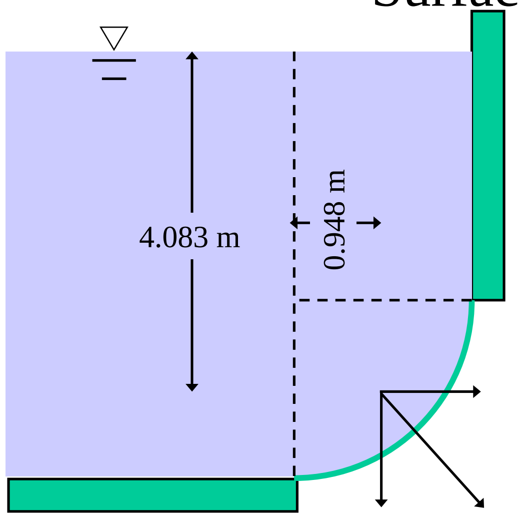


$I_{xc} = (1\text{ m})(2\text{ m})^3/12 = 0.667\text{ m}^4$

$y_c = 4\text{ m}$

$$y_R = \frac{0.667\text{ m}^4}{(4\text{ m})(2\text{ m})(1\text{ m})} + (4\text{ m}) = 4.083\text{ m}$$

Example: Forces on Curved Surfaces

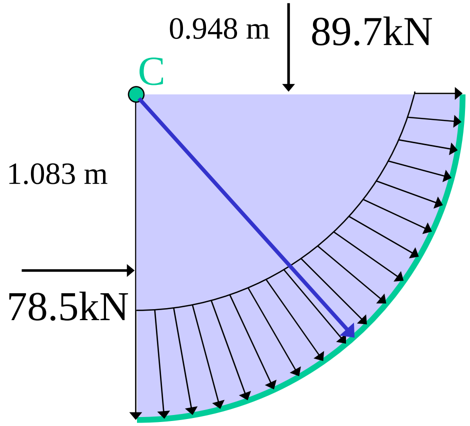


78.5 kN horizontal

89.7 kN vertical

119.2 kN resultant

Cylindrical Surface Force Check



- All pressure forces pass through point C.
- The pressure force applies no moment about point C.
- The resultant must pass through point C.

(78.5kN)(1.083m) - (89.7kN)(0.948m) = _0_

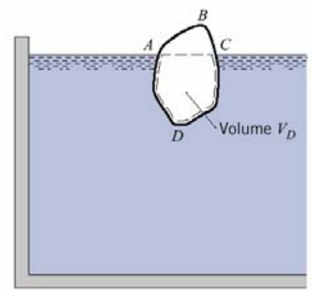
Static Surface Forces Summary

- Forces caused by gravity (or total acceleration) on submerged surfaces
  - horizontal surfaces (normal to total acceleration) $F_R = \gamma hA$ — Location where $p = p_{ref}$
  - inclined surfaces (y coordinate has origin at free surface) $F_R = \gamma h_c A$ $y_R = \frac{I_{xc}}{y_c A} + y_c$
  - curved surfaces
    - Horizontal component $F_R = \gamma h_c A$
    - Vertical component (weight of fluid above surface)
  -

Buoyant Force

- The resultant force exerted on a body by a ***static fluid*** in which it is fully or partially submerged
  - The projection of the body on a vertical plane is always _zero_.
  - The vertical components of pressure on the top and bottom surfaces are _different_

Archimedes Principle



- Archimedes Principle

  F_B = weight displaced fluid

- Line of action passes through the centroid of displaced volume

Buoyant Force: Line of Action

- The buoyant force acts through the centroid of the displaced volume of fluid (center of buoyancy)

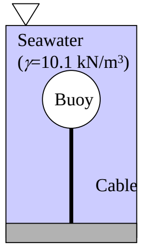
$$\gamma \int_{\forall} x\, d\forall = \gamma \forall x_c \quad x_c = \frac{1}{\forall}\int_{\forall} x\, d\forall$$

$\forall$ = volume

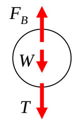
$\gamma d\forall$ = distributed force

x_c = centroid of volume

Example



Spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the sea floor with a cable as shown. The buoy normally floats on the surface, at other times the water depth increases so that the buoy is completely immersed as shown. What is the tension in the cable?



$$\sum_y F_y = 0 = F_B - W - T$$

$$F_B = \gamma\forall = \gamma \frac{\pi}{6} d^3 = (10{,}100\, N/m^3)\frac{\pi}{6}(1.5\,m)^3 = 17{,}850\, N$$

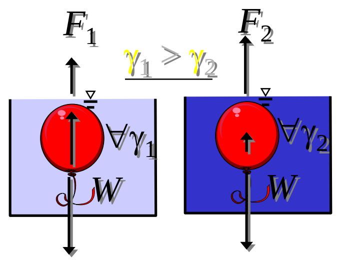
$$T = F_B - W$$

$$= 17{,}850 - 8{,}500\, N$$

$$= 9{,}350\, N$$

Buoyant Force: Applications

- Using buoyancy it is possible to determine:
  - Weight of an object
  - Volume of an object
  - Specific gravity of an object

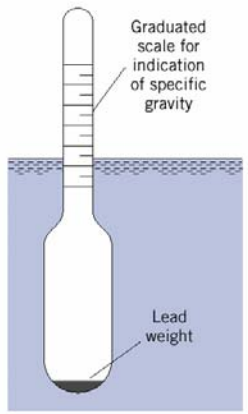


Force balance

$F_1 + \forall\gamma_1 = W \quad F_2 + \forall\gamma_2 = W$

(With F_1, F_2, γ_1, and γ_2 given)

Hydrometer

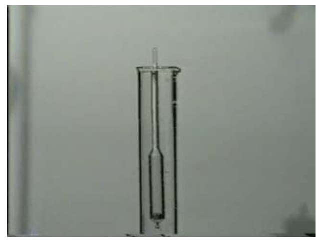


- Buoyant force

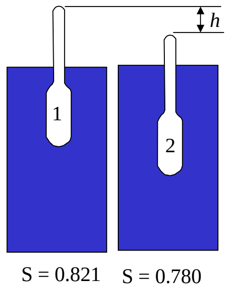
  F_B = weight of the hydrometer

  must remain constant

- Hydrometer floats deeper or shallower depending on the specific weight of the fluid



Example



A hydrometer weighs 0.0216 N and has a stem at the upper end that is cylindrical and 2.8 mm in diameter.

How much deeper will it float in oil of S=0.78 than in alcohol of S=0.821?

For position 1:

$$W_{hydrometer} = W_{displaced\ water}$$
$$0.0216 = 0.821 * 9810 * V_1$$
$$V_1 = 2.68x10^{-6}\ m^3$$

For position 2:

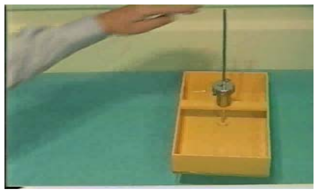
$$W_{hydrometer} = W_{displaced\ water}$$
$$0.0216 = 0.780 * 9810 * (V_1 + Ah)$$
$$= 0.780 * 9810 * [2.68x10^{-6} + \frac{\pi}{4}(0.0028)^2 h]$$
$$\boxed{h = 0.0232\,m = 23.2\,mm}$$

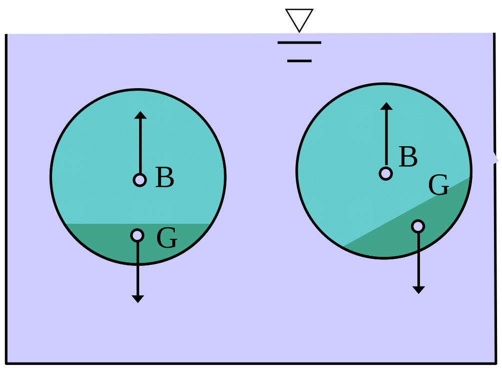
A sailboat is sailing on Caspian sea. The captain is in a hurry to get to shore and decides to cut the anchor off and toss it overboard to lighten the boat. Does the water level of Caspian sea ~~increase~~ or decrease?

Why? The anchor displaces less water when it is lying on the bottom of the lake than it did when in the boat.

Rotational Stability of Submerged Bodies

- A completely submerged body is stable when its center of gravity is below the center of buoyancy





Exercise:

2.89, 2.84, 2.95, 2.104